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**Supplemental tables and R codes for the paper
“Asymptotic cumulants of the minimum phi-divergence estimator for
categorical data under possible model misspecification”**

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This article supplements Ogasawara (2019a, b).

Part 1 gives numerical results under correct model specification in Tables S2.1 to S2.13. In the tables, “.00” indicates a rounded value of zero up to the second place while “0” indicates an exactly zero value.

Part 2 gives R codes and examples of running the programs.

References

- Ogasawara, H. (2019a). Asymptotic cumulants of the minimum phi-divergence estimator for categorical data under possible model misspecification. To appear in *Communications in Statistics – Theory and Methods*.
- Ogasawara, H. (2019b). Supplement to the paper “Asymptotic cumulants of the minimum phi-divergence estimator for categorical data under possible model misspecification”. To appear in *Economic Review (Otaru University of Commerce)*, 70 (2 & 3).
<http://www.res.otaru-uc.ac.jp/~emt-hogasa/>, <https://barrel.repo.nii.ac.jp/>.

Part 1. Numerical results under correct model specification

Table S2.1. True probabilities under correct model misspecification

Case	τ'			
A: The genetics of plants (Fisher, 1970)	.6000	.1500	.1500	.1000
B: 3-category truncated Poisson variate (Bishop et al., 1975)	.3679	.3679	.2642	
C: 4-category truncated Poisson variate	.2231	.3347	.2510	.1912
D: 4-category redundant truncated Poisson variate	.2231	.3347	.2510	.1912

Table S2.2. Simulated and theoretical standard errors multiplied by $n^{1/2}$ for the $M\phi$ Es when models are true: $\beta_2^{1/2}$

Case	Parameter	Sim.(n)			Sim.(n)			Th.
		(50)	(200)	(800)	(50)	(200)	(800)	
		$\lambda = 0$ (G^2 , ML)			$\lambda = -1$ (GM^2)			
A	θ	.801	.803	.801	.838	.812	.803	.800
B	θ	1.078	1.068	1.068	1.100	1.073	1.069	1.066
C	θ	1.286	1.271	1.278	1.339	1.283	1.280	1.272
D	θ_1	1.399	1.399	1.397	1.439	1.408	1.399	1.395
	θ_2	.418	.424	.422	.424	.425	.423	.423
		$\lambda = -2$ (Neyman)			$\lambda = 2/3$ (C-R)			
A	θ	.890	.826	.807	.792	.801	.801	.800
B	θ	1.452	1.078	1.070	1.066	1.064	1.067	1.066
C	θ	2.614	1.298	1.284	1.263	1.265	1.276	1.272
D	θ_1	1.499	1.419	1.402	1.381	1.394	1.396	1.395
	θ_2	.432	.427	.423	.416	.423	.422	.423
		$\lambda = 1$ (X^2 , Pearson)			$\lambda = 2$			
A	θ	.790	.800	.801	.791	.801	.801	.800
B	θ	1.059	1.063	1.067	1.041	1.058	1.066	1.066
C	θ	1.254	1.262	1.275	1.237	1.257	1.274	1.272
D	θ_1	1.374	1.392	1.396	1.360	1.388	1.395	1.395
	θ_2	.414	.423	.422	.412	.422	.422	.423

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_2^{1/2}$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.2. (continued)

Case	$n: 50$		$n: 200$		$n: 800$	
	Z	NC	Z	NC	Z	NC
A	579	95	0	0	0	0
B	0	10	0	0	0	0
C	3	9	0	0	0	0
D	3	3251	0	2	0	0

Note. n = the number of observations, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence.

Table S2.3. Simulated and theoretical ratios of the higher- and lower-order asymptotic standard errors for the $M\phi$ Es when models are true

Case	n	50		200		800		50		200		800	
Parameter		Sim.	Th.	Sim.	Th.	Sim.	Th.	Sim.	Th.	Sim.	Th.	Sim.	Th.
$\lambda = 0 (G^2, ML)$						$\lambda = -1 (GM^2)$							
A	θ	1.001	1.015	1.004	1.004	1.002	1.001	1.047	1.053	1.015	1.014	1.004	1.003
B	θ	1.012	.981	1.002	.995	1.002	.999	1.032	.999	1.006	1.000	1.003	1.000
C	θ	1.011	.975	.999	.994	1.004	.998	1.052	1.010	1.008	1.003	1.006	1.001
D	θ_1	1.002	.986	1.003	.996	1.001	.999	1.031	1.010	1.009	1.003	1.003	1.001
	θ_2	.990	.990	1.003	.998	.999	.999	1.003	1.006	1.006	1.002	1.000	1.000
$\lambda = -2 (Neyman)$						$\lambda = 2/3 (C-R)$							
A	θ	1.113	1.113	1.032	1.029	1.008	1.007	.990	1.004	1.001	1.001	1.001	1.000
B	θ	1.362	1.017	1.011	1.004	1.004	1.001	1.000	.969	.999	.992	1.001	.998
C	θ	2.055	1.052	1.020	1.013	1.009	1.003	.992	.955	.994	.989	1.003	.997
D	θ_1	1.075	1.039	1.017	1.010	1.005	1.002	.990	.972	.999	.993	1.001	.998
	θ_2	1.022	1.024	1.010	1.006	1.001	1.002	.983	.981	1.001	.995	.999	.999
$\lambda = 1 (X^2, Pearson)$						$\lambda = 2$							
A	θ	.988	1.002	1.000	1.001	1.001	1.000	.989	1.014	1.002	1.004	1.002	1.001
B	θ	.994	.962	.997	.991	1.001	.998	.977	.943	.993	.986	1.000	.997
C	θ	.985	.947	.992	.987	1.003	.997	.973	.925	.988	.982	1.001	.996
D	θ_1	.985	.965	.998	.991	1.000	.998	.975	.950	.995	.988	.999	.997
	θ_2	.980	.976	1.000	.994	.998	.999	.973	.963	.998	.991	.998	.998

Note. n = the number of observations, Sim. = simulated value = SD/ASE, SD = the standard deviation from simulation, $ASE = n^{-1/2} \bar{\beta}_2^{1/2}$, Th. = theoretical value = HASE/ASE, $HASE = (n^{-1} \beta_2 + n^{-2} \beta_{\Delta 2})^{1/2}$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.4. Simulated and theoretical biases multiplied by n for the $M\phi$ Es when models are true: β_1

Case	Parameter	Sim.(n)			Th.	Sim.(n)			Th.
		(50)	(200)	(800)		(50)	(200)	(800)	
		$\lambda = 0$ (G^2 , ML)				$\lambda = -1$ (GM^2)			
A	θ	.09	-.03	.12	0	.30	.16	.30	.18
B	θ	.22	.24	.09	0	.23	.25	.09	0
C	θ	.17	.24	.06	0	.19	.26	.08	.02
D	θ_1	.89	.59	.35	0	1.45	1.08	.83	.48
	θ_2	-.01	.06	-.01	-.06	.09	.15	.08	.03
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	.52	.35	.48	.36	-.01	-.14	-.00	-.12
B	θ	.27	.25	.09	0	.22	.24	.09	0
C	θ	.36	.26	.09	.03	.16	.23	.05	-.01
D	θ_1	2.08	1.59	1.32	.96	.56	.27	.03	-.32
	θ_2	.20	.24	.17	.12	-.07	-.00	-.01	-.12
		$\lambda = 1$ (X^2 , Pearson)				$\lambda = 2$			
A	θ	-.04	-.19	-.06	-.18	-.11	-.32	-.22	-.36
B	θ	.22	.24	.09	0	.21	.24	.09	0
C	θ	.15	.23	.05	-.02	.16	.22	.04	-.03
D	θ_1	.40	.11	-.12	-.48	-.03	-.36	-.60	-.96
	θ_2	-.10	-.03	-.10	-.15	-.18	-.12	-.19	-.24

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_1 , G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic. The "0" indicates an exactly zero value.

Table S2.5. Simulated and theoretical skewnesses multiplied by $n^{1/2}$ for the $M\phi$ Es when models are true: $\beta_3 / \beta_2^{3/2}$

Case		Sim.(n)				Sim.(n)				
Parameter		(50)	(200)	(800)	Th.	(50)	(200)	(800)	Th.	
		$\lambda = 0$ (G^2 , ML)					$\lambda = -1$ (GM^2)			
A	θ	.37	.05	-.30	.11	.60	.11	-.30	.11	
B	θ	1.78	1.68	1.95	.42	1.81	1.69	1.95	.42	
C	θ	1.30	1.33	1.29	.43	1.33	1.33	1.31	.43	
D	θ_1	2.53	2.12	2.10	-.12	2.74	2.16	2.11	-.12	
	θ_2	1.47	1.06	1.24	.98	1.55	1.07	1.24	.98	
		$\lambda = -2$ (Neyman)					$\lambda = 2/3$ (C-R)			
A	θ	1.18	.30	-.26	.11	.42	.07	-.29	.11	
B	θ	317	1.69	1.95	.42	1.78	1.68	1.95	.42	
C	θ	343	1.31	1.32	.43	1.28	1.33	1.29	.43	
D	θ_1	3.38	2.28	2.14	-.12	2.53	2.11	2.09	-.12	
	θ_2	1.71	1.09	1.25	.98	1.46	1.07	1.24	.98	
		$\lambda = 1$ (X^2 , Pearson)					$\lambda = 2$			
A	θ	.45	.08	-.29	.11	.52	.13	-.27	.11	
B	θ	1.78	1.68	1.95	.42	1.78	1.68	1.95	.42	
C	θ	1.27	1.33	1.28	.43	1.19	1.32	1.27	.43	
D	θ_1	2.55	2.12	2.09	-.12	2.63	2.14	2.09	-.12	
	θ_2	1.46	1.07	1.24	.98	1.48	1.08	1.24	.98	

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_3 / \beta_2^{3/2}$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.6. Simulated and theoretical kurtoses multiplied by n for the $M\phi$ Es when models are true: β_4 / β_2^2

Case		Sim.(n)				Sim.(n)				
Parameter		(50)	(200)	(800)	Th.	(50)	(200)	(800)	Th.	
		$\lambda = 0 (G^2, ML)$					$\lambda = -1 (GM^2)$			
A	θ	-8.1	-4.1	13.9	-.4	-8.3	-3.3	13.5	-.4	
B	θ	7.2	5.9	18.6	-8.5	7.8	6.2	18.6	-8.5	
C	θ	3.9	7.0	6.2	-10.4	5.8	7.0	6.9	-10.4	
D	θ_1	11.7	12.2	6.5	-7.7	15.3	12.9	7.8	-10.3	
	θ_2	-2.9	3.9	14.4	-.2	-2.1	3.7	14.7	-2.4	
		$\lambda = -2 (Neyman)$					$\lambda = 2/3 (C-R)$			
A	θ	-2.8	.0	13.3	-.4	-7.1	-4.2	14.2	-.4	
B	θ	2.5e5	6.9	18.6	-8.5	7.4	5.8	18.6	-8.5	
C	θ	1.7e5	8.1	7.7	-10.4	4.0	7.2	5.8	-10.4	
D	θ_1	32.4	15.2	9.4	-12.8	11.7	12.0	5.9	-6.0	
	θ_2	.5	3.7	15.0	-4.6	-3.0	4.0	14.2	1.3	
		$\lambda = 1 (X^2, Pearson)$					$\lambda = 2$			
A	θ	-6.8	-4.2	14.3	-.4	-6.4	-4.1	14.5	-.4	
B	θ	7.7	5.9	18.6	-8.5	9.1	6.1	18.7	-8.5	
C	θ	4.0	7.3	5.7	-10.4	3.1	7.4	5.1	-10.4	
D	θ_1	11.9	12.0	5.5	-5.1	12.2	12.0	4.7	-2.5	
	θ_2	-3.0	4.1	14.1	2.0	-2.8	4.3	13.8	4.2	

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_4 / β_2^2 , G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, $x \text{ e } y = x10^y$.

Table S2.7. Simulated and squared biases and added higher-order asymptotic biases multiplied by n^2 for the $M\phi$ Es when models are true: β_1^2 and $\beta_{\Delta 2}$

Case	Parameter	β_1^2				$\beta_{\Delta 2}$			
		Sim.(n)			Th.	Sim.(n)			Th.= $\beta_{\Delta 2}$
		(50)	(200)	(800)		(50)	(200)	(800)	
$\lambda = -2$ (Neyman)									
A	θ	.27	.12	.23	.13	7.6	8.4	8.4	7.6
B	θ	.07	.06	.01	0	48.6	5.2	7.7	1.9
C	θ	.13	.07	.01	.00	261	13.2	23.7	8.6
D	θ_1	4.31	2.54	1.73	.91	15.0	13.2	14.3	7.7
	θ_2	.04	.06	.03	.01	.4	.7	.2	.4
$\lambda = 2/3$ (C-R)									
A	θ	.00	.02	.00	.01	-.7	.2	1.1	.2
B	θ	.05	.06	.01	0	-.0	-.6	2.1	-3.5
C	θ	.02	.05	.00	.00	-1.2	-3.7	7.9	-7.1
D	θ_1	.31	.07	.00	.10	-2.0	-.6	1.9	-5.4
	θ_2	.00	.00	.00	.01	-.3	.1	-.4	-.3
$\lambda = 2$									
A	θ	.01	.10	.05	.13	-.7	.4	1.7	.9
B	θ	.05	.06	.01	0	-2.6	-3.4	-.7	-6.2
C	θ	.02	.05	.00	.00	-4.4	-7.8	3.6	-11.6
D	θ_1	.00	.13	.36	.91	-4.8	-4.1	-1.7	-9.6
	θ_2	.03	.02	.04	.06	-.5	-.1	-.6	-.6

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_1^2 or $\beta_{\Delta 2}$, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic. The "0" indicates an exactly zero value.

Table S2.8. Simulated and theoretical standard errors of the studentized $M\phi$ Es when models are true: $\beta_2^{1/2}$,

Case		Sim.(n)				Sim.(n)				
Parameter		(50)	(200)	(800)	Th.	(50)	(200)	(800)	Th.	
		$\lambda = 0$ (G^2 , ML)					$\lambda = -1$ (GM^2)			
A	θ	1.079	1.021	1.005	1	1.093	1.025	1.006	1	
B	θ	1.033	1.006	1.003	1	1.033	1.006	1.003	1	
C	θ	1.035	1.004	1.006	1	1.038	1.005	1.006	1	
D	θ_1	.991	1.002	1.002	1	.987	1.002	1.002	1	
	θ_2	1.036	1.015	1.002	1	1.034	1.015	1.002	1	
		$\lambda = -2$ (Neyman)					$\lambda = 2/3$ (C-R)			
A	θ	1.074	1.021	1.005	1	1.058	1.015	1.004	1	
B	θ	1.859	1.006	1.003	1	1.030	1.006	1.003	1	
C	θ	1.4e6	1.003	1.006	1	1.024	1.002	1.005	1	
D	θ_1	.974	1.000	1.001	1	.989	1.002	1.002	1	
	θ_2	1.030	1.014	1.002	1	1.035	1.015	1.002	1	
		$\lambda = 1$ (X^2 , Pearson)					$\lambda = 2$			
A	θ	1.049	1.011	1.003	1	1.028	.999	1.000	1	
B	θ	1.027	1.006	1.003	1	1.014	1.005	1.003	1	
C	θ	1.018	1.001	1.005	1	1.005	.995	1.004	1	
D	θ_1	.989	1.002	1.002	1	.990	1.000	1.001	1	
	θ_2	1.034	1.015	1.002	1	1.033	1.014	1.002	1	

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_2^{1/2}$, $\lambda=1$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, $x \text{ e } y = x10^y$.

Table S2.9. Simulated and theoretical biases multiplied by $n^{1/2}$ for the studentized $M\phi$ Es when models are true: β_1'

Case	Parameter	Sim.(n)			Th.	Sim.(n)			Th.
		(50)	(200)	(800)		(50)	(200)	(800)	
		$\lambda = 0 (G^2, ML)$				$\lambda = -1 (GM^2)$			
A	θ	.03	-.10	.09	-.06	.26	.13	.32	.17
B	θ	-.48	-.43	-.57	-.21	-.48	-.43	-.57	-.21
C	θ	-.40	-.32	-.46	-.21	-.39	-.31	-.45	-.20
D	θ_1	-.03	-.26	-.43	.06	.31	.09	-.08	.40
	θ_2	-.60	-.44	-.58	-.64	-.38	-.22	-.36	-.42
		$\lambda = -2 (Neyman)$				$\lambda = 2/3 (C-R)$			
A	θ	.35	.31	.53	.39	-.13	-.25	-.06	-.20
B	θ	-.42	-.43	-.57	-.21	-.48	-.43	-.57	-.21
C	θ	3.2e4	-.30	-.44	-.19	-.39	-.32	-.47	-.22
D	θ_1	.59	.41	.25	.74	-.29	-.49	-.66	-.17
	θ_2	-.17	-.01	-.15	-.21	-.75	-.58	-.72	-.78
		$\lambda = 1 (X^2, Pearson)$				$\lambda = 2$			
A	θ	-.18	-.32	-.13	-.28	-.30	-.49	-.33	-.50
B	θ	-.47	-.43	-.57	-.21	-.46	-.43	-.57	-.21
C	θ	-.38	-.32	-.47	-.23	-.33	-.31	-.48	-.24
D	θ_1	-.42	-.61	-.77	-.28	-.79	-.97	-1.12	-.63
	θ_2	-.83	-.66	-.79	-.85	-1.04	-.87	-1.01	-1.07

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_1' , G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, $x e y = x10^y$.

Table S2.10. Simulated and theoretical skewnesses multiplied by $n^{1/2}$ for the studentized $M\phi$ Es when models are true: β_3'

Case	Parameter	Sim.(n)			Th.	Sim.(n)			Th.
		(50)	(200)	(800)		(50)	(200)	(800)	
		$\lambda = 0 (G^2, \text{ML})$				$\lambda = -1 (GM^2)$			
A	θ	-.37	-.42	-.63	-.22	-.36	-.39	-.62	-.22
B	θ	-2.42	-2.31	-2.06	-.84	-2.43	-2.31	-2.06	-.84
C	θ	-1.97	-1.77	-1.79	-.85	-1.97	-1.77	-1.78	-.85
D	θ_1	-1.57	-2.01	-1.98	.24	-1.59	-2.01	-1.98	.24
	θ_2	-1.98	-2.46	-2.15	-1.96	-1.98	-2.47	-2.15	-1.96
		$\lambda = -2 (\text{Neyman})$				$\lambda = 2/3 (\text{C-R})$			
A	θ	-.71	-.53	-.66	-.22	-.62	-.48	-.64	-.22
B	θ	747	-2.31	-2.06	-.84	-2.36	-2.31	-2.05	-.84
C	θ	2236	-1.73	-1.77	-.85	-1.86	-1.75	-1.80	-.85
D	θ_1	-1.98	-2.10	-2.00	.24	-1.74	-2.06	-2.00	.24
	θ_2	-2.25	-2.51	-2.16	-1.96	-2.03	-2.47	-2.15	-1.96
		$\lambda = 1 (X^2, \text{Pearson})$				$\lambda = 2$			
A	θ	-.72	-.51	-.64	-.22	-1.00	-.59	-.65	-.22
B	θ	-2.32	-2.31	-2.05	-.84	-2.18	-2.29	-2.05	-.84
C	θ	-1.82	-1.74	-1.80	-.85	-1.74	-1.68	-1.79	-.85
D	θ_1	-1.85	-2.09	-2.01	.24	-2.27	-2.20	-2.04	.24
	θ_2	-2.06	-2.47	-2.15	-1.96	-2.16	-2.50	-2.16	-1.96

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_3' , G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.11. Proportions of a population value below the one-sided confidence intervals when models are true: $n = 50$ and $\lambda = -2$ (Neyman's statistic)

Case		Nominal values						
Parameter	Method	.0050	.0250	.1000	.5000	.9000	.9750	.9950
A	θ	Z = 48, NC = 10						
	NF	.0240	.0590	.1546	.5299	.8794	.9507	.9815
	NR	.0054	.0207	.0858	.5299	.9142	.9660	.9840
	C-F	.0085	.0073	.0494	.4984	.9413	.9830	.9934
	Hall	.0003	.0064	.0487	.4986	.9439	.9844	.9949
B	θ	Z = 0, NC = 0						
	NF	.0026	.0198	.1003	.4992	.8655	.9455	.9828
	NR	.0039	.0233	.0936	.4992	.8753	.9508	.9821
	C-F	.0035	.0175	.0827	.5011	.9001	.9670	.9846
	Hall	.0031	.0171	.0827	.5011	.9001	.9670	.9846
C	θ	Z = 0, NC = 0						
	NF	.0050	.0300	.1106	.5039	.8596	.9415	.9794
	NR	.0035	.0189	.0825	.5039	.8940	.9621	.9867
	C-F	.0023	.0163	.0777	.5056	.9179	.9744	.9898
	Hall	.0019	.0155	.0776	.5061	.9183	.9752	.9900
D	θ_1	Z = 0, NC = 367						
	NF	.0062	.0325	.1218	.5458	.9097	.9711	.9937
	NR	.0014	.0169	.1043	.5458	.9116	.9707	.9909
	C-F	.0023	.0104	.0684	.5115	.9079	.9722	.9918
	Hall	.0011	.0096	.0684	.5115	.9082	.9722	.9919
	θ_2							
	NF	.0041	.0215	.0992	.5116	.8844	.9609	.9859
	NR	.0019	.0131	.0931	.5116	.8893	.9637	.9857
	C-F	.0021	.0131	.0896	.5020	.9098	.9798	.9954
	Hall	.0020	.0128	.0896	.5020	.9108	.9831	.9970

Note. NF = the normal approximation by the Fisher information matrix, NR = the normal approximation by the robust ASE estimate, C-F = the Cornish-Fisher expansion, Hall = Hall's (1992) monotonic cubic transformation, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted case(s) due to non-convergence.

Table S2.12. Proportions of a population value below the one-sided confidence intervals when models are true: $n = 50$ and $\lambda = 2/3$ (the Cressie-Read statistic)

Case		Nominal values						
Parameter	Method	.0050	.0250	.1000	.5000	.9000	.9750	.9950
A	θ	Z = 48, NC = 10						
	NF	.0105	.0317	.1125	.5048	.8833	.9606	.9873
	NR	.0127	.0380	.1096	.5048	.8827	.9553	.9831
	C-F	.0048	.0274	.1074	.5161	.9053	.9701	.9871
	Hall	.0042	.0240	.1074	.5161	.9054	.9847	.9893
B	θ	Z = 0, NC = 0						
	NF	.0016	.0147	.0798	.4992	.8872	.9604	.9875
	NR	.0016	.0169	.0831	.4992	.8831	.9584	.9852
	C-F	.0021	.0189	.0841	.4992	.8949	.9628	.9902
	Hall	.0021	.0189	.0841	.4992	.8949	.9629	.9904
C	θ	Z = 0, NC = 0						
	NF	.0016	.0173	.0879	.5053	.8894	.9642	.9899
	NR	.0025	.0192	.0895	.5053	.8863	.9592	.9874
	C-F	.0034	.0214	.1004	.5058	.8899	.9646	.9899
	Hall	.0034	.0213	.1004	.5058	.8904	.9653	.9906
D	θ_1	Z = 0, NC = 367						
	NF	.0022	.0156	.0832	.5030	.9000	.9689	.9932
	NR	.0026	.0168	.0856	.5030	.8944	.9660	.9930
	C-F	.0006	.0138	.0855	.5196	.8947	.9710	.9949
	Hall	.0006	.0122	.0855	.5196	.8947	.9710	.9949
	θ_2							
	NF	.0020	.0154	.0766	.4715	.8728	.9554	.9843
	NR	.0025	.0162	.0790	.4715	.8746	.9544	.9836
	C-F	.0038	.0217	.0987	.5001	.9045	.9800	.9955
	Hall	.0037	.0217	.0987	.5001	.9049	.9833	.9988

Note. NF = the normal approximation by the Fisher information matrix, NR = the normal approximation by the robust ASE estimate, C-F = the Cornish-Fisher expansion, Hall = Hall's (1992) monotonic cubic transformation, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence.

Table S2.13. Proportions of a population value below the one-sided confidence intervals when models are true: $n = 50$ and $\lambda = 2$

Case		Nominal values						
Parameter	Method	.0050	.0250	.1000	.5000	.9000	.9750	.9950
A	θ	Z = 48, NC = 10						
	NF	.0091	.0347	.1067	.4945	.8771	.9589	.9862
	NR	.0132	.0318	.0973	.4945	.8893	.9514	.9827
	C-F	.0084	.0290	.0906	.5167	.8966	.9566	.9885
	Hall	.0052	.0238	.0892	.5168	.9004	.9606	.9901
B	θ	Z = 0, NC = 0						
	NF	.0015	.0145	.0778	.4992	.8895	.9619	.9883
	NR	.0024	.0191	.0832	.4992	.8823	.9584	.9827
	C-F	.0041	.0234	.0932	.4992	.8754	.9468	.9730
	Hall	.0041	.0234	.0932	.4992	.8754	.9472	.9744
C	θ	Z = 0, NC = 0						
	NF	.0010	.0149	.0832	.5041	.8939	.9660	.9908
	NR	.0048	.0224	.0871	.5041	.8947	.9601	.9932
	C-F	.0058	.0272	.1005	.5056	.8903	.9642	.9845
	Hall	.0051	.0271	.1005	.5065	.8904	.9674	.9859
D	θ_1	Z = 0, NC = 367						
	NF	.0022	.0118	.0699	.4779	.8908	.9643	.9928
	NR	.0014	.0133	.0757	.4779	.8900	.9538	.9895
	C-F	.0013	.0156	.0832	.5164	.8996	.9681	.9899
	Hall	.0012	.0154	.0832	.5164	.8996	.9681	.9899
	θ_2							
	NF	.0018	.0135	.0706	.4510	.8690	.9531	.9829
	NR	.0021	.0134	.0675	.4510	.8533	.9564	.9834
	C-F	.0046	.0236	.0993	.5109	.9132	.9683	.9932
	Hall	.0046	.0232	.0993	.5109	.9137	.9725	.9947

Note. NF = the normal approximation by the Fisher information matrix, NR = the normal approximation by the robust ASE estimate, C-F = the Cornish-Fisher expansion, Hall = Hall's (1992) monotonic cubic transformation, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence.

Part 2. R codes and examples of running the programs

In Part 2, two examples are shown. In Example 1, log-linear models in a $2 \times 2 \times 2$ contingency table are analyzed using the program “mpe3a.R”. In Example 2, log-linear models including the linear-by-linear model in a two-way contingency table are dealt with by the program “mpe2a.R”. In these programs the function “mpese” is used.

Example 1.

The program “mpe3a.R”

```
#
# Robust asymptotic standard errors of the minimizing phi-divergence
# estimators (MPEs) for log-linear models in a 2x2x2 table
# under possible model misspecification
#
# <mpe3a.R> April 16, 2018
#
# Use as: R prompt >source('file_name').
# The results will be given in 'r.u'.
#

starttime=proc.time()
sink ('r.u')
print(date(),quote=F)

##### function mpese
mpese=function(n,pr,al,w,qo,iq,ik,im,imax){
if(im==1){
print(' ',quote=F)
print('methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))',quote=F)
print('.....',quote=F)
}
delta=1e-8
q=qo
inv=0
se=rep(0,iq)
ser=rep(0,iq)

for (it in 1:(imax+1)){
```

```

if(it>imax){
print(paste('!!!! no convergence, the number of iterations=',
           it,sep=' '),quote=F)
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

pi=exp(w%%q)
pi=pi[,1,drop=T] # vectorized for later use
                # as 'diag(pi,nrow=length(pi))'

pi=pi/sum(pi)
wp=t(w)%%pi
wp=wp[,1,drop=T] # vectorized for later use
                # as 'wp%o%wp+...'

pq=matrix(0,ik,iq)
for (i in 1:ik){
for (j in 1:iq){
pq[i,j]=(w[i,j]-wp[j])*pi[i]
};}

pq2=array(0,c(ik,iq,iq))
for (i in 1:ik){
for (j1 in 1:iq){
for (j2 in 1:iq){
pq2[i,j1,j2]=(w[i,j1]-wp[j1])*pq[i,j2]
for (i1 in 1:ik){
pq2[i,j1,j2]=pq2[i,j1,j2]-w[i1,j1]*pq[i1,j2]*pi[i]
};};};}

g=rep(0,iq)
h=matrix(0,iq,iq)

for (i in 1:iq){
if(im==1)g[i]=-sum(pr/pi*pq[,i]) # note the minus sign
if(im==2)g[i]=sum((log(pi/pr)+1)*pq[,i])
if(im>=3 && im<=8)g[i]=sum(-a1[im-2]*(pr/pi)^(a1[im-2]+1)*pq[,i])

for (j in 1:i){
if(im==1)h[i,j]=-sum(-pr/pi^2*pq[,i]*pq[,j]+pr/pi*pq2[,i,j]) # note the
                                                                # minus sign

```



```

if(im==2)h[i,j]=sum(1/pi*pq[,i]*pq[,j]+(log(pi/pr)+1)*pq2[,i,j])
if(im>=3 && im<=8)h[i,j]=sum(al[im-2]*(al[im-2]+1)*pr^(al[im-2]+1)
/pi^(al[im-2]+2)*pq[,i]*pq[,j]-al[im-2]*(pr/pi)^(al[im-2]+1)*pq2[,i,j])

h[j,i]=h[i,j]
};}

aaa=eigen(h,symmetric=T)[['values']] # not -h
if(min(abs(aaa))<delta){
print(paste('!!!! The following Hessian is singular!, iteration=',
it,sep=''),quote=F)
print(h)
print('eigenvalues',quote=F)
print(aaa)
inv=1
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

h1=h
h=solve(h)

if(max(abs(g))<delta){

fi=-wp%o%wp+t(w)%*%diag(pi,nrow=length(pi))%*%w
fi=solve(fi)
se=sqrt(fi[row(fi)==col(fi)]/n)

gp=matrix(0,iq,ik)

for (i in 1:iq){
for (k in 1:ik){
if(im==1)gp[i,k]=-1/pi[k]*pq[k,i] # note the minus sign
if(im==2)gp[i,k]=-1/pr[k]*pq[k,i]
if(im>=3 && im<=8)gp[i,k]=-al[im-2]*(al[im-2]+1)*
pr[k]^al[im-2]/pi[k]^(al[im-2]+1)*pq[k,i]
};}

qp=-h%*%gp
qpi=qp%*%pr

```

```

qpi=qpi[,1,drop=T] # vectorized

fir=-qpi%o%qpi+qp%*%diag(pr,nrow=length(pr))%*%t(qp)
ser=sqrt(fir[row(fir)==col(fir)]/n)

q=as.vector(q) # vectorized for ease of presentation

return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))

} # end of if(max(abs(g))<delta)

iout=0
if(iout==1){
print(paste('##### iteration=',it,sep=' '),quote=F)
print('q(parameters),g(gradients)',quote=F)
print(q)
print(g)
print('Hessian',quote=F)
print(h1)
}

q=q-h%*%g
q=as.vector(q) # vectorized for ease of presentation

} # end of it-loop
} # end of function mpese
#####

al=c(-2,-0.5,0.5,2/3,1,2) # powers in power divergences
# except those for G^2(ML) and GM^2

cat('¥n
Robust asymptotic standard errors of the minimizing phi-divergence ¥n
estimators (MPEs) for log-linear models in a 2x2x2 table ¥n
under possible model misspecification ¥n
for a survey on alcohol(A), cigarette(C) and marijuana(M) use ¥n
for US high school seniors using the data ¥n
by Agresti (2013, p.346) ¥n')

```

```

ik=8 # the number of categories
fn=c(911,538,44,456,3,43,2,279) # a vectorized contingency table
                                # of Agresti (2013, p.346)

n=sum(fn)
pr=fn/n
ft=array(0,c(2,2,2),dimnames=list(c('A-Yes','A-No'),
                                c('C-Yes','C-No'),c('M-Yes','M-No')))

ijk=0
for (i in 1:2){;for (j in 1:2){;for (k in 1:2){
ijk=ijk+1
ft[i,j,k]=fn[ijk] };};}

print(' ',quote=F)
print(paste('total frequency=',n,sep=' '),quote=F)
print('frequencies',quote=F)
print(ft)
print('percentages',quote=F)
print(round(ft/n*100,1))

ws1=c(1,1,1,1,1,1,1, # The vectorized design matrix
      1,1,0,1,0,0,0, # for the saturated model
      1,0,1,0,1,0,0,
      1,0,0,0,0,0,0,
      0,1,1,0,0,1,0,
      0,1,0,0,0,0,0,
      0,0,1,0,0,0,0,
      0,0,0,0,0,0,0)

ws=matrix(data=ws1,8,7,byrow=T)
print('The K(the number of catogories) by q(the number of parameters)',quote=F)
print('design matrix for the saturated model',quote=F)
print(ws)

an=c('Model 1 (A,C,M)', 'Model 2 (A,CM)', 'Model 3 (C,AM)', 'Model 4 (M,AC)',
     'Model 5 (AC,AM)', 'Model 6 (AC,CM)', 'Model 7 (AM,CM)',
     'Model 8 (AC,AM,CM)', 'Model 9 (ACM)(saturated model)')

wd1=c(1,1,1,0,0,0,0, # Model 1

```

```

1,1,1,0,0,1,0, # Model 2
1,1,1,0,1,0,0, # Model 3
1,1,1,1,0,0,0, # Model 4
1,1,1,1,1,0,0, # Model 5
1,1,1,1,0,1,0, # Model 6
1,1,1,0,1,1,0, # Model 7
1,1,1,1,1,1,0, # Model 8
1,1,1,1,1,1,1) # Model 9
wd=matrix(wd1,9,7,byrow=T,dimnames=list(paste('Model ',1:9,sep=''),
paste('p-use ',1:7,sep='')))
print(' ',quote=F)
print('p-use=the pattern of prameter use (1:use,0:not use)',quote=F)
print(wd)

imax=200

for (mdl in 1:9){
print(' ',quote=F)
print('=====',quote=F)
print(an[mdl],quote=F)
iw=wd[mdl,]
iq=sum(iw)
print(paste('number of parameters=',iq,sep=''),qoute=F)
print('the pattern of parameter use (1;use,0:not use)',quote=F)
print(wd[mdl,])

iw=iw*1:7
iw=iw[iw>0]
w=ws[,iw]
print('the (reduced) design matrix for the model',quote=F)
print(w)

qo=rep(0,iq) # arbitrary initial values for MLEs

for (im in 1:8){
print('.....',quote=F)

abc=mpese(n,pr,al,w,qo,iq,ik,im,imax)

```

```
print(paste('estimation method=',im,sep=''),quote=F)
```

```
itr=abc[['it']]  
inv=abc[['inv']]  
q=abc[['qhat']]  
se=abc[['se']]  
ser=abc[['ser']]
```

```
print('parameter estimates',quote=F)  
print(q)  
print('information-based ASEs',quote=F)  
print(se)  
print('robust ASEs(RASEs)',quote=F)  
print(ser)  
print('RASE/ASE',quote=F)  
print(round(ser/se,3))
```

```
if(im==1)qo=q  
} # end of im-loop  
} # end of mdl-loop
```

```
print(proc.time()-starttime,quote=F)  
print(date(),quote=F)  
sink()
```

Results using the program “mpe3a.R”

[1] Thu Dec 06 10:42:19 2018

Robust asymptotic standard errors of the minimizing phi-divergence

estimators (MPEs) for log-linear models in a 2x2x2 table

under possible model misspecification

for a survey on alcohol(A), cigarette(C) and marijuana(M) use

for US high school seniors using the data

by Agresti (2013, p.346)

[1]

[1] total frequency=2276

[1] frequencies

, , M-Yes

	C-Yes	C-No
A-Yes	911	44
A-No	3	2

, , M-No

	C-Yes	C-No
A-Yes	538	456
A-No	43	279

[1] percentages

, , M-Yes

	C-Yes	C-No
A-Yes	40.0	1.9
A-No	0.1	0.1

, , M-No

	C-Yes	C-No
A-Yes	23.6	20.0
A-No	1.9	12.3

[1] The K(the number of categories) by q(the number of parameters)

[1] design matrix for the saturated model

[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]
[1,]	1	1	1	1	1	1

```
[2,] 1 1 0 1 0 0 0
[3,] 1 0 1 0 1 0 0
[4,] 1 0 0 0 0 0 0
[5,] 0 1 1 0 0 1 0
[6,] 0 1 0 0 0 0 0
[7,] 0 0 1 0 0 0 0
[8,] 0 0 0 0 0 0 0
```

```
[1]
```

```
[1] p-use=the pattern of parameter use (1:use,0:not use)
```

```
      p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
Model 1      1      1      1      0      0      0      0
Model 2      1      1      1      0      0      1      0
Model 3      1      1      1      0      1      0      0
Model 4      1      1      1      1      0      0      0
Model 5      1      1      1      1      1      0      0
Model 6      1      1      1      1      0      1      0
Model 7      1      1      1      0      1      1      0
Model 8      1      1      1      1      1      1      0
Model 9      1      1      1      1      1      1      1
```

```
[1]
```

```
[1] =====
```

```
[1] Model 1 (A,C,M)
```

```
[1] "number of parameters=3"
```

```
[1] the pattern of parameter use (1;use,0:not use)
```

```
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      0      0      0      0
```

```
[1] the (reduced) design matrix for the model
```

```
      [,1] [,2] [,3]
[1,] 1 1 1
[2,] 1 1 0
[3,] 1 0 1
[4,] 1 0 0
[5,] 0 1 1
[6,] 0 1 0
[7,] 0 0 1
[8,] 0 0 0
```

```
[1] .....
```

```
[1]
```

```
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
```

```
[1] .....
```

```
[1] estimation method=1
```

```
[1] parameter estimates
```

```
[1] 1.7851115 0.6493063 -0.3154188
```

```
[1] information-based ASEs
```

```
[1] 0.05975941 0.04415095 0.04244461
```

```
[1] robust ASEs(RASEs)
```

```
[1] 0.05975941 0.04415095 0.04244461
```

```
[1] RASE/ASE
```

```
[1] 1 1 1
```

```
[1] .....
```

```

[1] estimation method=2
[1] parameter estimates
[1] 3.62070082 1.47526026 -0.08688913
[1] information-based ASEs
[1] 0.13155570 0.05385366 0.04196175
[1] robust ASEs(RASEs)
[1] 0.28268430 0.11488380 0.08171046
[1] RASE/ASE
[1] 2.149 2.133 1.947
[1] .....
[1] !!!! The following Hessian is singular!, iteration=6
           [,1]      [,2]      [,3]
[1,] 9.491213e-34 2.794404e-18 2.794404e-18
[2,] 2.794404e-18 -5.547459e-16 1.432653e-16
[3,] 2.794404e-18 1.432653e-16 2.096498e-201
[1] eigenvalues
[1] 3.513827e-17 -3.171054e-19 -5.895671e-16
[1] estimation method=3
[1] parameter estimates
[1] 42.02769 36.50708 232.45397
[1] information-based ASEs
[1] 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 2.4459052 0.9183479 -0.3430608
[1] information-based ASEs
[1] 0.07737919 0.04641983 0.04254043
[1] robust ASEs(RASEs)
[1] 0.09906469 0.06414934 0.05833772
[1] RASE/ASE
[1] 1.280 1.382 1.371
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 1.4513710 0.5302778 -0.2114558
[1] information-based ASEs
[1] 0.05345371 0.04340437 0.04215671
[1] robust ASEs(RASEs)
[1] 0.05608016 0.03719939 0.03748794
[1] RASE/ASE
[1] 1.049 0.857 0.889
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 1.3764022 0.5053881 -0.1793792

```



```

[1] information-based ASEs
[1] 0.05224785 0.04326777 0.04209091
[1] robust ASEs(RASEs)
[1] 0.05582415 0.03591637 0.03668945
[1] RASE/ASE
[1] 1.068 0.830 0.872
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 1.2585712 0.4680354 -0.1230820
[1] information-based ASEs
[1] 0.05050034 0.04307535 0.04200159
[1] robust ASEs(RASEs)
[1] 0.05564573 0.03415946 0.03560498
[1] RASE/ASE
[1] 1.102 0.793 0.848
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] 1.046233324 0.407716979 -0.005356147
[1] information-based ASEs
[1] 0.04779021 0.04279631 0.04192233
[1] robust ASEs(RASEs)
[1] 0.05589151 0.03184063 0.03395304
[1] RASE/ASE
[1] 1.170 0.744 0.810
[1]
[1] =====
[1] Model 2 (A,CM)
[1] "number of parameters=4"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      0      0      1      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4]
[1,]    1    1    1    1
[2,]    1    1    0    0
[3,]    1    0    1    0
[4,]    1    0    0    0
[5,]    0    1    1    1
[6,]    0    1    0    0
[7,]    0    0    1    0
[8,]    0    0    0    0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] 1.7851115 -0.2351197 -2.7712291 3.2243089

```

```

[1] information-based ASEs
[1] 0.05975941 0.05551319 0.15198577 0.16098117
[1] robust ASEs(RASEs)
[1] 0.05975941 0.05551319 0.15198577 0.16098117
[1] RASE/ASE
[1] 1 1 1 1
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] 3.40224851 0.09977517 -2.42208118 2.84599037
[1] information-based ASEs
[1] 0.11869392 0.05700172 0.14465901 0.15323224
[1] robust ASEs(RASEs)
[1] 0.28544261 0.06544265 0.15681141 0.16392242
[1] RASE/ASE
[1] 2.405 1.148 1.084 1.070
[1] .....
[1] !!!! The following Hessian is singular!, iteration=3
           [,1]      [,2]      [,3]      [,4]
[1,] -0.0696625479  0.0001115558 -0.0019400632 -0.0019400632
[2,]  0.0001115558 -0.0046929124 -0.0001389803 -0.0001389803
[3,] -0.0019400632 -0.0001389803 -0.0764020430 -0.0764020430
[4,] -0.0019400632 -0.0001389803 -0.0764020430 -0.0764020430
[1] eigenvalues
[1] 3.252607e-19 -4.692447e-03 -6.957232e-02 -1.528948e-01
[1] estimation method=3
[1] parameter estimates
[1] 4.211925 7.435819 -62.668405 58.062392
[1] information-based ASEs
[1] 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 2.3010654 -0.1056044 -2.6500693 3.0413680
[1] information-based ASEs
[1] 0.07286796 0.05533851 0.14825942 0.15710751
[1] robust ASEs(RASEs)
[1] 0.08363237 0.05943445 0.15441332 0.16271760
[1] RASE/ASE
[1] 1.148 1.074 1.042 1.036
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 1.5332992 -0.2750535 -2.7966206 3.2919142
[1] information-based ASEs
[1] 0.05485746 0.05587158 0.15307516 0.16213350

```

```

[1] robust ASEs(RASEs)
[1] 0.05592170 0.05299905 0.15184018 0.16106101
[1] RASE/ASE
[1] 1.019 0.949 0.992 0.993
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 1.4757195 -0.2787266 -2.7965153 3.2999774
[1] information-based ASEs
[1] 0.05386142 0.05595584 0.15313835 0.16220649
[1] robust ASEs(RASEs)
[1] 0.05524171 0.05238165 0.15205009 0.16127635
[1] RASE/ASE
[1] 1.026 0.936 0.993 0.994
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 1.3832318 -0.2787648 -2.7909086 3.3050369
[1] information-based ASEs
[1] 0.05235464 0.05608109 0.15307457 0.16215053
[1] robust ASEs(RASEs)
[1] 0.05422613 0.05143172 0.15244168 0.16167401
[1] RASE/ASE
[1] 1.036 0.917 0.996 0.997
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] 1.2075357 -0.2562814 -2.7614993 3.2862846
[1] information-based ASEs
[1] 0.04979820 0.05627964 0.15247260 0.16152366
[1] robust ASEs(RASEs)
[1] 0.05213885 0.04987298 0.15274596 0.16206301
[1] RASE/ASE
[1] 1.047 0.886 1.002 1.003
[1]
[1] =====
[1] Model 3 (C,AM)
[1] "number of parameters=4"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      0      1      0      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4]
[1,]    1    1    1    1
[2,]    1    1    0    0
[3,]    1    0    1    1
[4,]    1    0    0    0
[5,]    0    1    1    0
[6,]    0    1    0    0
[7,]    0    0    1    0

```

```

[8,]  0  0  0  0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] 1.1271857 0.6493063 -4.1651136 4.1250877
[1] information-based ASEs
[1] 0.06412196 0.04415095 0.45067236 0.45294452
[1] robust ASEs(RASEs)
[1] 0.06412196 0.04415095 0.45067235 0.45294450
[1] RASE/ASE
[1] 1 1 1 1
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] 2.106684 1.347147 -3.132103 3.067634
[1] information-based ASEs
[1] 0.09145557 0.05179729 0.42236054 0.42457389
[1] robust ASEs(RASEs)
[1] 0.1387445 0.1129191 0.4999101 0.4984764
[1] RASE/ASE
[1] 1.517 2.180 1.184 1.174
[1] .....
[1] !!!! The following Hessian is singular!, iteration=8
           [,1]      [,2]      [,3]      [,4]
[1,] -3.981748e-07 -9.525347e-20 3.140589e-07 -8.411584e-08
[2,] -9.525347e-20 2.184960e+02 -1.263444e-14 -1.572626e-19
[3,] 3.140589e-07 -1.263444e-14 -3.468130e-07 -9.613071e-17
[4,] -8.411584e-08 -1.572626e-19 -9.613071e-17 -8.411584e-08
[1] eigenvalues
[1] 2.184960e+02 -9.958067e-09 -1.252174e-07 -6.939282e-07
[1] estimation method=3
[1] parameter estimates
[1] 2.2605520 0.4054651 24.0481213 -25.3655070
[1] information-based ASEs
[1] 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 1.5103048 0.8576832 -3.7655976 3.6203349
[1] information-based ASEs
[1] 0.07126124 0.04583647 0.42872928 0.43103321
[1] robust ASEs(RASEs)
[1] 0.07919432 0.05908070 0.45573468 0.45743781

```

```

[1] RASE/ASE
[1] 1.111 1.289 1.063 1.061
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 0.9148656 0.5475605 -4.3852091 4.4504034
[1] information-based ASEs
[1] 0.06163574 0.04350317 0.46946715 0.47170316
[1] robust ASEs(RASEs)
[1] 0.06432889 0.03929165 0.45086202 0.45348532
[1] RASE/ASE
[1] 1.044 0.903 0.960 0.961
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 0.8668759 0.5246981 -4.4348982 4.5284530
[1] information-based ASEs
[1] 0.06121019 0.04337316 0.47448745 0.47671443
[1] robust ASEs(RASEs)
[1] 0.06477607 0.03857981 0.45095993 0.45364686
[1] RASE/ASE
[1] 1.058 0.889 0.950 0.952
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.7926992 0.4886477 -4.5116900 4.6526624
[1] information-based ASEs
[1] 0.06065383 0.04317967 0.48289679 0.48510947
[1] robust ASEs(RASEs)
[1] 0.06562382 0.03784033 0.45105026 0.45380997
[1] RASE/ASE
[1] 1.082 0.876 0.934 0.935
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] 0.6686971 0.4225198 -4.6399334 4.8724512
[1] information-based ASEs
[1] 0.06003456 0.04286117 0.49912282 0.50131064
[1] robust ASEs(RASEs)
[1] 0.06745441 0.03792329 0.45084606 0.45365032
[1] RASE/ASE
[1] 1.124 0.885 0.903 0.905
[1]
[1] =====
[1] Model 4 (M,AC)
[1] "number of parameters=4"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      1      0      0      0
[1] the (reduced) design matrix for the model

```

```

      [,1] [,2] [,3] [,4]
[1,]  1    1    1    1
[2,]  1    1    0    1
[3,]  1    0    1    0
[4,]  1    0    0    0
[5,]  0    1    1    0
[6,]  0    1    0    0
[7,]  0    0    1    0
[8,]  0    0    0    0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1]  0.5762534 -1.8097133 -0.3154188  2.8737341
[1] information-based ASEs
[1]  0.07455682 0.15905298 0.04244461 0.16729609
[1] robust ASEs(RASEs)
[1]  0.07455682 0.15905298 0.04244461 0.16729609
[1] RASE/ASE
[1]  1  1  1  1
[1] .....
[1] estimation method=2
[1] parameter estimates
[1]  1.6314458 -0.8720668 -0.2469873  2.2939163
[1] information-based ASEs
[1]  0.12066127 0.20322858 0.04224226 0.21037819
[1] robust ASEs(RASEs)
[1]  0.3602700 0.4377359 0.1096806 0.4124869
[1] RASE/ASE
[1]  2.986 2.154 2.596 1.961
[1] .....
[1] !!!! The following Hessian is singular!, iteration=13
      [,1]      [,2]      [,3]      [,4]
[1,] -2.193640e-06  1.493926e-08 -4.482037e-16 -2.178701e-06
[2,]  1.493926e-08 -4.308265e-07  1.077543e-17 -2.845620e-14
[3,] -4.482037e-16  1.077543e-17  6.032793e+00 -4.481966e-16
[4,] -2.178701e-06 -2.845620e-14 -4.481966e-16 -2.178701e-06
[1] eigenvalues
[1]  6.032793e+00 -7.194313e-09 -4.310606e-07 -4.364913e-06
[1] estimation method=3
[1] parameter estimates
[1] -3.326420  19.287532 -2.662588 -14.305043
[1] information-based ASEs
[1]  0  0  0  0
[1] robust ASEs(RASEs)
[1]  0  0  0  0
[1] RASE/ASE
[1] NaN NaN NaN NaN

```

```

[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 0.8099698 -1.6128774 -0.4033236 2.7746509
[1] information-based ASEs
[1] 0.08214896 0.16764309 0.04277751 0.17559115
[1] robust ASEs(RASEs)
[1] 0.08775606 0.17766999 0.05441635 0.18662614
[1] RASE/ASE
[1] 1.068 1.060 1.272 1.063
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 0.5128898 -1.8568149 -0.2313024 2.8302470
[1] information-based ASEs
[1] 0.07134895 0.15353539 0.04220285 0.16186388
[1] robust ASEs(RASEs)
[1] 0.07520578 0.16136632 0.03804372 0.16861910
[1] RASE/ASE
[1] 1.054 1.051 0.901 1.042
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 0.5049230 -1.8621036 -0.2074611 2.8086844
[1] information-based ASEs
[1] 0.07063586 0.15212139 0.04214793 0.16046066
[1] robust ASEs(RASEs)
[1] 0.07541919 0.16210747 0.03716219 0.16915711
[1] RASE/ASE
[1] 1.068 1.066 0.882 1.054
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.4967147 -1.8671780 -0.1656515 2.7660457
[1] information-based ASEs
[1] 0.06950300 0.14977871 0.04206606 0.15812974
[1] robust ASEs(RASEs)
[1] 0.07571069 0.16305092 0.03581640 0.16981202
[1] RASE/ASE
[1] 1.089 1.089 0.851 1.074
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] 0.49162992 -1.86987983 -0.07685628 2.66703317
[1] information-based ASEs
[1] 0.06741072 0.14529992 0.04195314 0.15366765
[1] robust ASEs(RASEs)
[1] 0.07597795 0.16377327 0.03323355 0.17001995
[1] RASE/ASE
[1] 1.127 1.127 0.792 1.106

```

```

[1]
[1] =====
[1] Model 5 (AC,AM)
[1] "number of parameters=5"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      1      1      0      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    1    1    1
[2,]    1    1    0    1    0
[3,]    1    0    1    0    1
[4,]    1    0    0    0    0
[5,]    0    1    1    0    0
[6,]    0    1    0    0    0
[7,]    0    0    1    0    0
[8,]    0    0    0    0    0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] -0.08167244 -1.80971327 -4.16511363  2.87373412  4.12508776
[1] information-based ASEs
[1] 0.0780971 0.1590530 0.4506724 0.1672961 0.4529445
[1] robust ASEs(RASEs)
[1] 0.08341921 0.15905298 0.45067237 0.16729609 0.45294453
[1] RASE/ASE
[1] 1.068 1.000 1.000 1.000 1.000
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] -0.7432232 -1.8479935 -4.6283525  3.4903467  4.6904931
[1] information-based ASEs
[1] 0.0834269 0.1519561 0.5322889 0.1641620 0.5342567
[1] robust ASEs(RASEs)
[1] 0.1347046 0.1634865 0.6219219 0.1946395 0.6254193
[1] RASE/ASE
[1] 1.615 1.076 1.168 1.186 1.171
[1] .....
[1] !!!! The following Hessian is singular!, iteration=7
      [,1]      [,2]      [,3]      [,4]      [,5]
[1,] -1.290155e-05  9.618684e-07  9.799231e-09 -7.536576e-11 -2.284777e-12
[2,]  9.618684e-07 -2.039938e-05 -7.282805e-10 -1.943744e-05  2.465179e-12
[3,]  9.799231e-09 -7.282805e-10 -5.990683e-07  2.352082e-12 -5.892667e-07
[4,] -7.536576e-11 -1.943744e-05  2.352082e-12 -1.943744e-05  2.294826e-12
[5,] -2.284777e-12  2.465179e-12 -5.892667e-07  2.294826e-12 -5.892667e-07
[1] eigenvalues
[1] -4.876704e-09 -4.387827e-07 -1.183450e-06 -1.291990e-05 -3.937970e-05

```



```

[1] estimation method=3
[1] parameter estimates
[1] 13.637221 -2.518663 -7.181809 -10.630457 -9.463397
[1] information-based ASEs
[1] 0 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN NaN
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] -0.3138298 -1.8316042 -4.4321792 3.1076961 4.3858503
[1] information-based ASEs
[1] 0.07909004 0.15581548 0.49880917 0.16527515 0.50088362
[1] robust ASEs(RASEs)
[1] 0.09265513 0.16148501 0.51695891 0.17351200 0.51961269
[1] RASE/ASE
[1] 1.172 1.036 1.036 1.050 1.037
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 0.05913331 -1.79188405 -3.89367230 2.71348876 3.89470083
[1] information-based ASEs
[1] 0.07803361 0.16146867 0.40392857 0.16903949 0.40644343
[1] robust ASEs(RASEs)
[1] 0.08080089 0.15687250 0.48345300 0.16417623 0.48525264
[1] RASE/ASE
[1] 1.035 0.972 1.197 0.971 1.194
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 0.09352406 -1.78807130 -3.81292069 2.67219709 3.82910849
[1] information-based ASEs
[1] 0.07808069 0.16216416 0.39080111 0.16957493 0.39339542
[1] robust ASEs(RASEs)
[1] 0.08031267 0.15609411 0.49947647 0.16326557 0.50116052
[1] RASE/ASE
[1] 1.029 0.963 1.278 0.963 1.274
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.1491953 -1.7835243 -3.6709297 2.6039786 3.7164104
[1] information-based ASEs
[1] 0.07820985 0.16344721 0.36870460 0.17059956 0.37144640
[1] robust ASEs(RASEs)
[1] 0.07957022 0.15461894 0.52480521 0.16162130 0.52633070
[1] RASE/ASE
[1] 1.017 0.946 1.423 0.947 1.417
[1] .....

```

```

[1] estimation method=8
[1] parameter estimates
[1] 0.2515631 -1.7843301 -3.3809681 2.4756117 3.4972851
[1] information-based ASEs
[1] 0.07863296 0.16658667 0.32769003 0.17325901 0.33076344
[1] robust ASEs(RASEs)
[1] 0.07827543 0.15198863 0.55261916 0.15870356 0.55395581
[1] RASE/ASE
[1] 0.995 0.912 1.686 0.916 1.675
[1]
[1] =====
[1] Model 6 (AC,CM)
[1] "number of parameters=5"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      1      0      1      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5]
[1,] 1 1 1 1 1
[2,] 1 1 0 1 0
[3,] 1 0 1 0 0
[4,] 1 0 0 0 0
[5,] 0 1 1 0 1
[6,] 0 1 0 0 0
[7,] 0 0 1 0 0
[8,] 0 0 0 0 0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] 0.5762534 -2.6941394 -2.7712291 2.8737341 3.2243089
[1] information-based ASEs
[1] 0.07455682 0.16257385 0.15198577 0.16729609 0.16098117
[1] robust ASEs(RASEs)
[1] 0.07455682 0.16867486 0.15198577 0.16729609 0.16098117
[1] RASE/ASE
[1] 1.000 1.038 1.000 1.000 1.000
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] 0.5867064 -3.7661408 -3.2674343 3.9190234 3.7592763
[1] information-based ASEs
[1] 0.07476418 0.25666594 0.19054343 0.25963278 0.19784751
[1] robust ASEs(RASEs)
[1] 0.07893518 0.38500977 0.27880730 0.37952182 0.28431162
[1] RASE/ASE
[1] 1.056 1.500 1.463 1.462 1.437
[1] .....

```

```

[1] estimation method=3
[1] parameter estimates
[1] 0.5300749 -4.5844949 -4.0438607 4.7404595 4.5628822
[1] information-based ASEs
[1] 0.07452326 0.36693100 0.27659477 0.36908580 0.28168682
[1] robust ASEs(RASEs)
[1] 0.0827107 0.5770305 0.6525872 0.5741721 0.6548953
[1] RASE/ASE
[1] 1.110 1.573 2.359 1.556 2.325
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 0.589673 -3.135789 -2.961575 3.304283 3.425468
[1] information-based ASEs
[1] 0.0746865 0.1955481 0.1654277 0.1994526 0.1737497
[1] robust ASEs(RASEs)
[1] 0.07634255 0.20669261 0.16909582 0.20400869 0.17761494
[1] RASE/ASE
[1] 1.022 1.057 1.022 1.023 1.022
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 0.5612236 -2.4482495 -2.6570020 2.6309742 3.1138446
[1] information-based ASEs
[1] 0.07444836 0.14701924 0.14466211 0.15224486 0.15408738
[1] robust ASEs(RASEs)
[1] 0.07405539 0.16765607 0.15252408 0.16815772 0.16125501
[1] RASE/ASE
[1] 0.995 1.140 1.054 1.105 1.047
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 0.556848 -2.393284 -2.629807 2.576190 3.088984
[1] information-based ASEs
[1] 0.07441907 0.14378091 0.14299525 0.14912270 0.15252645
[1] robust ASEs(RASEs)
[1] 0.07402519 0.16750570 0.15293945 0.16860471 0.16161049
[1] RASE/ASE
[1] 0.995 1.165 1.070 1.131 1.060
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.5492365 -2.3084704 -2.5864044 2.4911810 3.0505026
[1] information-based ASEs
[1] 0.07436986 0.13894437 0.14039411 0.14446742 0.15009749
[1] robust ASEs(RASEs)
[1] 0.07404363 0.16702877 0.15358176 0.16917652 0.16217897
[1] RASE/ASE
[1] 0.996 1.202 1.094 1.171 1.080
[1] .....

```

```

[1] estimation method=8
[1] parameter estimates
[1] 0.5336424 -2.1638004 -2.5083417 2.3445205 2.9851442
[1] information-based ASEs
[1] 0.07427613 0.13112717 0.13589607 0.13696303 0.14591926
[1] robust ASEs(RASEs)
[1] 0.07427614 0.16568140 0.15455259 0.17003058 0.16310429
[1] RASE/ASE
[1] 1.000 1.264 1.137 1.241 1.118
[1]
[1] =====
[1] Model 7 (AM,CM)
[1] "number of parameters=5"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      0      1      1      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5]
[1,] 1 1 1 1 1
[2,] 1 1 0 0 0
[3,] 1 0 1 1 0
[4,] 1 0 0 0 0
[5,] 0 1 1 0 1
[6,] 0 1 0 0 0
[7,] 0 0 1 0 0
[8,] 0 0 0 0 0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] 1.1271857 -0.2351197 -6.6209236 4.1250875 3.2243089
[1] information-based ASEs
[1] 0.06412196 0.05551319 0.47371264 0.45294446 0.16098117
[1] robust ASEs(RASEs)
[1] 0.06412196 0.05551319 0.49045848 0.45294440 0.16098117
[1] RASE/ASE
[1] 1.000 1.000 1.035 1.000 1.000
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] 1.3866267 -0.2416015 -6.7152864 4.2072392 3.2622248
[1] information-based ASEs
[1] 0.07006960 0.05645955 0.54223632 0.52434585 0.16024436
[1] robust ASEs(RASEs)
[1] 0.08956285 0.06642767 0.58682340 0.56031784 0.16773061
[1] RASE/ASE
[1] 1.278 1.177 1.082 1.069 1.047
[1] .....

```

```

[1] estimation method=3
[1] parameter estimates
[1] 1.8303925 -0.0723899 -6.2842636 3.8415783 3.0994144
[1] information-based ASEs
[1] 0.08244917 0.05694385 0.55372571 0.53659661 0.15823225
[1] robust ASEs(RASEs)
[1] 0.1670253 0.0841692 0.6275273 0.6002464 0.1758406
[1] RASE/ASE
[1] 2.026 1.478 1.133 1.119 1.111
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 1.2398450 -0.2539314 -6.7515457 4.2382149 3.2643174
[1] information-based ASEs
[1] 0.06658249 0.05600861 0.51959139 0.50069074 0.16103558
[1] robust ASEs(RASEs)
[1] 0.07130090 0.05997769 0.53645793 0.50753728 0.16423027
[1] RASE/ASE
[1] 1.071 1.071 1.032 1.014 1.020
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 1.0368174 -0.2030364 -6.3573661 3.8954172 3.1628244
[1] information-based ASEs
[1] 0.06232453 0.05507772 0.41464904 0.39110344 0.16015877
[1] robust ASEs(RASEs)
[1] 0.06193008 0.05301371 0.57731932 0.52125495 0.16031239
[1] RASE/ASE
[1] 0.994 0.963 1.392 1.333 1.001
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 1.0107805 -0.1914056 -6.2615709 3.8115026 3.1420482
[1] information-based ASEs
[1] 0.06183944 0.05495403 0.39637964 0.37180503 0.15987018
[1] robust ASEs(RASEs)
[1] 0.06172081 0.05254417 0.61758515 0.55671323 0.16034489
[1] RASE/ASE
[1] 0.998 0.956 1.558 1.497 1.003
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.9641650 -0.1683224 -6.0830305 3.6543965 3.1041640
[1] information-based ASEs
[1] 0.06100846 0.05473966 0.36524822 0.33862770 0.15943861
[1] robust ASEs(RASEs)
[1] 0.06164986 0.05197494 0.67821597 0.61152081 0.16005457
[1] RASE/ASE
[1] 1.011 0.949 1.857 1.806 1.004
[1] .....

```

```

[1] estimation method=8
[1] parameter estimates
[1] 0.8603861 -0.1084225 -5.7126489 3.3258146 3.0251793
[1] information-based ASEs
[1] 0.05932593 0.05430971 0.31019036 0.27844951 0.15928362
[1] robust ASEs(RASEs)
[1] 0.06219327 0.05166710 0.72958879 0.65836744 0.15845056
[1] RASE/ASE
[1] 1.048 0.951 2.352 2.364 0.995
[1]
[1] =====
[1] Model 8 (AC,AM,CM)
[1] "number of parameters=6"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      1      1      1      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6]
[1,] 1 1 1 1 1 1
[2,] 1 1 0 1 0 0
[3,] 1 0 1 0 1 0
[4,] 1 0 0 0 0 0
[5,] 0 1 1 0 0 1
[6,] 0 1 0 0 0 0
[7,] 0 0 1 0 0 0
[8,] 0 0 0 0 0 0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] 0.487719 -1.886669 -5.309042 2.054534 2.986014 2.847889
[1] information-based ASEs
[1] 0.0757672 0.1626970 0.4751970 0.1740643 0.4646780 0.1638394
[1] robust ASEs(RASEs)
[1] 0.07578959 0.16422965 0.48432895 0.17565960 0.46773287 0.16352188
[1] RASE/ASE
[1] 1.000 1.009 1.019 1.009 1.007 0.998
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] 0.488160 -1.884680 -5.345490 2.052237 3.020513 2.850064
[1] information-based ASEs
[1] 0.07577525 0.16264703 0.48244025 0.17403877 0.47198534 0.16402133
[1] robust ASEs(RASEs)
[1] 0.07582508 0.16430701 0.49894061 0.17581954 0.48237421 0.16403440
[1] RASE/ASE
[1] 1.001 1.010 1.034 1.010 1.022 1.000
[1] .....

```

```

[1] estimation method=3
[1] parameter estimates
[1] 0.4886384 -1.8824705 -5.3779379 2.0496918 3.0508872 2.8523821
[1] information-based ASEs
[1] 0.07578315 0.16257455 0.48889114 0.17398931 0.47848148 0.16420149
[1] robust ASEs(RASEs)
[1] 0.07588175 0.16454615 0.53138496 0.17619115 0.51303139 0.16478538
[1] RASE/ASE
[1] 1.001 1.012 1.087 1.013 1.072 1.004
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 0.4879297 -1.8857265 -5.3276442 2.0534444 3.0036786 2.8489344
[1] information-based ASEs
[1] 0.07577119 0.16267610 0.47889498 0.17405568 0.46841072 0.16392916
[1] robust ASEs(RASEs)
[1] 0.07580485 0.16425456 0.48832981 0.17572004 0.47200461 0.16374754
[1] RASE/ASE
[1] 1.000 1.010 1.020 1.010 1.008 0.999
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 0.487538 -1.887461 -5.289985 2.055452 2.967778 2.846976
[1] information-based ASEs
[1] 0.07576342 0.16270743 0.47140271 0.17406279 0.46084303 0.16375502
[1] robust ASEs(RASEs)
[1] 0.07577665 0.16419849 0.48805353 0.17559809 0.47084925 0.16333279
[1] RASE/ASE
[1] 1.000 1.009 1.035 1.009 1.022 0.997
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 0.4874857 -1.8876847 -5.2835860 2.0557127 2.9616197 2.8467083
[1] information-based ASEs
[1] 0.07576222 0.16270830 0.47012675 0.17405977 0.45955217 0.16372856
[1] robust ASEs(RASEs)
[1] 0.07577229 0.16418059 0.49101098 0.17556991 0.47358366 0.16327215
[1] RASE/ASE
[1] 1.000 1.009 1.044 1.009 1.031 0.997
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.4873947 -1.8880663 -5.2707852 2.0561576 2.9492452 2.8462355
[1] information-based ASEs
[1] 0.07575993 0.16270589 0.46757118 0.17404979 0.45696473 0.16367868
[1] robust ASEs(RASEs)
[1] 0.07576292 0.16412775 0.49927527 0.17549491 0.48145360 0.16314772
[1] RASE/ASE
[1] 1.000 1.009 1.068 1.008 1.054 0.997
[1] .....

```

```

[1] estimation method=8
[1] parameter estimates
[1] 0.4872396 -1.8886492 -5.2331209 2.0568476 2.9123670 2.8453705
[1] information-based ASEs
[1] 0.07575415 0.16266522 0.46002223 0.17398853 0.44930482 0.16355740
[1] robust ASEs(RASEs)
[1] 0.0757276 0.1638260 0.5378484 0.1751104 0.5200499 0.1627182
[1] RASE/ASE
[1] 1.000 1.007 1.169 1.006 1.157 0.995
[1]
[1] =====
[1] Model 9 (ACM)(saturated model)
[1] "number of parameters=7"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1      1      1      1      1      1      1
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,] 1 1 1 1 1 1 1
[2,] 1 1 0 1 0 0 0
[3,] 1 0 1 0 1 0 0
[4,] 1 0 0 0 0 0 0
[5,] 0 1 1 0 0 1 0
[6,] 0 1 0 0 0 0 0
[7,] 0 0 1 0 0 0 0
[8,] 0 0 0 0 0 0 0
[1] .....
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408

```



```

[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=3
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs

```

```

[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] 0.4912810 -1.8700117 -4.9380646 2.0353774 2.5997614 2.2754768
[7] 0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
    user system elapsed
    2.31 0.04 2.35
[1] Thu Dec 06 10:42:21 2018

```

The program “mpe2a.R”

```
#
# Robust asymptotic standard errors of the minimizing phi-divergence
# estimators (MPEs) for log-linear models including the linear-by-
# linear model in a two-way contingency table with two sets
# of ordinal categories under possible model misspecification
#
# <mpe2a.R> April 16, 2018
#
# Use as: R prompt >source('file_name').
# The results will be given in 'r.u'.
#

starttime=proc.time()
sink ('r.u')
print(date(),quote=F)

##### function mpese
mpese=function(n,pr,a1,w,qo,iq,ik,im,imax){
  if(im==1){
    print('methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))',quote=F)
    print('.....',quote=F)
  }
  delta=1e-8
  q=qo
  inv=0
  se=rep(0,iq)
  ser=rep(0,iq)

  for (it in 1:(imax+1)){
    if(it>imax){
      print(paste('!!!! no convergence, the number of iterations=',
                  it,sep=''),quote=F)
      return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
    }

    pi=exp(w%*%q)
```

```

pi=pi[,1,drop=T] # vectorized for later use
                # as 'diag(pi,nrow=length(pi))'
pi=pi/sum(pi)
wp=t(w)%*%pi
wp=wp[,1,drop=T] # vectorized for later use
                # as 'wp%o%wp+...'
pq=matrix(0,ik,iq)
for (i in 1:ik){
for (j in 1:iq){
pq[i,j]=(w[i,j]-wp[j])*pi[i]
};}

pq2=array(0,c(ik,iq,iq))
for (i in 1:ik){
for (j1 in 1:iq){
for (j2 in 1:iq){
pq2[i,j1,j2]=(w[i,j1]-wp[j1])*pq[i,j2]
for (i1 in 1:ik){
pq2[i,j1,j2]=pq2[i,j1,j2]-w[i1,j1]*pq[i1,j2]*pi[i]
};};};}

g=rep(0,iq)
h=matrix(0,iq,iq)

for (i in 1:iq){
if(im==1)g[i]=-sum(pr/pi*pq[,i]) # note the minus sign
if(im==2)g[i]=sum((log(pi/pr)+1)*pq[,i])
if(im>=3 && im<=8)g[i]=sum(-al[im-2]*(pr/pi)^(al[im-2]+1)*pq[,i])

for (j in 1:i){
if(im==1)h[i,j]=-sum(-pr/pi^2*pq[,i]*pq[,j]+pr/pi*pq2[,i,j]) # note the
                                                                # minus sign
if(im==2)h[i,j]=sum(1/pi*pq[,i]*pq[,j]+(log(pi/pr)+1)*pq2[,i,j])
if(im>=3 && im<=8)h[i,j]=sum(al[im-2]*(al[im-2]+1)*pr^(al[im-2]+1)
/pi^(al[im-2]+2)*pq[,i]*pq[,j]-al[im-2]*(pr/pi)^(al[im-2]+1)*pq2[,i,j])

h[j,i]=h[i,j]
};}

```

```

aaa=eigen(h,symmetric=T)[['values']] # not -h
if(min(abs(aaa))<delta){
print(paste('!!!! The following Hessian is singular!, iteration=',
           it,sep=' '),quote=F)
print(h)
print('eigenvalues',quote=F)
print(aaa)
inv=1
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

h1=h
h=solve(h)

if(max(abs(g))<delta){

fi=-wp%o%wp+t(w)%%diag(pi,nrow=length(pi))%*%w
fi=solve(fi)
se=sqrt(fi[row(fi)==col(fi)]/n)

gp=matrix(0,iq,ik)

for (i in 1:iq){
for (k in 1:ik){
if(im==1)gp[i,k]=-1/pi[k]*pq[k,i] # note the minus sign
if(im==2)gp[i,k]=-1/pr[k]*pq[k,i]
if(im>=3 && im<=8)gp[i,k]=-al[im-2]*(al[im-2]+1)*
pr[k]^al[im-2]/pi[k]^(al[im-2]+1)*pq[k,i]
};}

qp=-h%*%gp
qpi=qp%*%pr
qpi=qpi[,1,drop=T] # vectorized

fir=-qpi%o%qpi+qp%*%diag(pr,nrow=length(pr))%*%t(qp)
ser=sqrt(fir[row(fir)==col(fir)]/n)

q=as.vector(q) # vectorized for ease of presentation

```

```

return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))

} # end of if(max(abs(g))<delta)

iout=0
if(iout==1){
print(paste('##### iteration=',it,sep=' '),quote=F)
print('q(parameters),g(gradients)',quote=F)
print(q)
print(g)
print('Hessian',quote=F)
print(h1)
}

q=q-h%*%g
q=as.vector(q) # vectorized for ease of presentation

} # end of it-loop
} # end of function mpese
#####

al=c(-2,-0.5,0.5,2/3,1,2) # powers in power divergences
                        # except those for G^2(ML) and GM^2

cat('¥n
Robust asymptotic standard errors of the minimizing phi-divergence ¥n
estimators (MPEs) for log-linear models including the linear-by- ¥n
linear model in a two-way contingency table with two sets ¥n
of ordinal categories under possible model misspecification ¥n
on opinions about premarital sex (X) and teenage birth control (Y) ¥n
using the data by Agresti (2013, p.386) ¥n')

i0=4 # the number of row categories
j0=4 # the number of column categories
ik=i0*j0 # the number of categories

fn=c(81, 68, 60, 38, # a vectorized contingency table
     24, 26, 29, 14, # of Agresti (2013, p.386)
     18, 41, 74, 42,

```

```

36, 57,161,157)
n=sum(fn)
pr=fn/n
ft=matrix(fn,4,4,byrow=T,dimnames=list(paste('X',1:4,sep=' '),
                                         paste('Y',1:4,sep=' ')))

print(' ',quote=F)
print(paste('total frequency=',n,sep=' '),quote=F)
print('frequencies',quote=F)
print(ft)
print('percentages',quote=F)
print(round(ft/n*100,1))

print('row marginal(%)',quote=F)
print(round(apply(ft/n*100,1,sum),1))
print('column marginal(%)',quote=F)
print(round(apply(ft/n*100,2,sum),1))

ws1=c(1,0,0,1,0,0,0,1,0,0,0,0,0,0,0, # The vectorized design
      1,0,0,0,1,0,0,0,1,0,0,0,0,0,0, # matrix for the linear-by-
      1,0,0,0,0,1,0,0,0,1,0,0,0,0,0, # linear and saturated models
      1,0,0,0,0,0,0,0,0,0,0,0,0,0,0, # (the 7th column is null,
      0,1,0,1,0,0,0,0,0,0,1,0,0,0,0, # which will be filled by
      0,1,0,0,1,0,0,0,0,0,0,1,0,0,0, # the linear-by-linear
      0,1,0,0,0,1,0,0,0,0,0,0,1,0,0, # model)
      0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
      0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,
      0,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,
      0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,1,
      0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
      0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,
      0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,
      0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

ws=matrix(data=ws1,16,16,byrow=T)

print(' ',quote=F)
print('The K(the number of catogories) by K',quote=F)

```

```

print('design matrix for the linear-by-linear and saturated models',
      quote=F)
print('(the 7th column is null, which will be filled by the',quote=F)
print('linear-by-linear model)',quote=F)
print(ws)

an=c('Model 1 (X,Y)', 'Model 2A (X,Y,linA)', 'Model 2B (X,Y,linB)',
     'Model 3 (XY)(saturated)')

wd1=c(1,1,1,1,1,1,0,0,0,0,0,0,0,0,0, # Model 1
      1,1,1,1,1,1,0,0,0,0,0,0,0,0,0, # Model 2A
      1,1,1,1,1,1,0,0,0,0,0,0,0,0,0, # Model 2B
      1,1,1,1,1,1,0,1,1,1,1,1,1,1,1) # Model 3(saturated)
wd=matrix(wd1,4,16,byrow=T,
          dimnames=list(an,paste('p-use ',1:16,sep='')))
print(' ',quote=F)
print('p-use=the pattern of parameter use (1:use,0:not use)',quote=F)
print(wd)
print(' ',quote=F)

ua=c(1,2,3,4)
va=ua
print('unstandardized u for Model (X,Y,linA)',quote=F)
print(ua)
print('unstandardized v for Model (X,Y,linA)',quote=F)
print(va)
print('unstandardized u*v¥' for Model (X,Y,linA)',quote=F)
print(round(ua%ova,3))

pr1=apply(ft/n,1,sum)
pr2=apply(ft/n,2,sum)
ua=(ua-sum(ua*pr1))/sqrt(sum(ua^2*pr1)-sum(ua*pr1)^2)
va=(va-sum(va*pr2))/sqrt(sum(va^2*pr2)-sum(va*pr2)^2)
print('standardized u for Model (X,Y,linA)',quote=F)
print(round(ua,3))
print('standardized v for Model (X,Y,linA)',quote=F)
print(round(va,3))
print('standardized u*v¥' for Model (X,Y,linA)',quote=F)
print(round(ua%ova,3))

```



```

ub=c(1,2,4,5)
vb=ub
print(' ',quote=F)
print('unstandadized u for Model (X,Y,linB)',quote=F)
print(ub)
print('unstandardized v for Model (X,Y,linB)',quote=F)
print(vb)
print('unstandardized u*v%' for Model (X,Y,linB)',quote=F)
print(round(ub%o%vb,3))

ub=(ub-sum(ub*pr1))/sqrt(sum(ub^2*pr1)-sum(ub*pr1)^2)
vb=(vb-sum(vb*pr2))/sqrt(sum(vb^2*pr2)-sum(vb*pr2)^2)
print('standardized u for Model (X,Y,linB)',quote=F)
print(round(ub,3))
print('standardized v for Model (X,Y,linB)',quote=F)
print(round(vb,3))
print('standardized u*v%' for Model (X,Y,linB)',quote=F)
print(round(ub%o%vb,3))

imax=200

for (mdl in 1:4){
print(' ',quote=F)
print('=====',quote=F)
print(an[mdl],quote=F)
iw=wd[mdl,]
iq=sum(iw)
print(paste('number of parameters=',iq,sep=''),qoute=F)
print('the pattern of parameter use (1;use,0:not use)',quote=F)
print(wd[mdl,])

if(mdl==2)ws[,i0+j0-1]=as.vector(va%o%ua) # not ua%o%va
if(mdl==3)ws[,i0+j0-1]=as.vector(vb%o%ub) # not ub%o%vb

iw=iw*1:ik
iw=iw[iw>0]
w=ws[,iw]
print('the (reduced) design matrix for the model',quote=F)

```

```

print(round(w,3))

qo=rep(0,iq) # arbitrary initial values for MLEs

for (im in 1:8){
print('.....',quote=F)

abc=mpese(n,pr,al,w,qo,iq,ik,im,imax)

print(paste('estimation method=',im,sep=''),quote=F)

itr=abc[['it']]
inv=abc[['inv']]
q=abc[['qhat']]
se=abc[['se']]
ser=abc[['ser']]

print('parameter estimates',quote=F)
print(q)
print('information-based ASEs',quote=F)
print(se)
print('robust ASEs(RASEs)',quote=F)
print(ser)
print('RASE/ASE',quote=F)
print(round(ser/se,3))

if(im==1)qo=q
} # end of im-loop
} # end of mdl-loop

print(proc.time()-starttime,quote=F)
print(date(),quote=F)
sink()

```

Results using the program “mpe2a.R”

[1] Thu Dec 06 10:38:49 2018

Robust asymptotic standard errors of the minimizing phi-divergence

estimators (MPEs) for log-linear models including the linear-by-

linear model in a two-way contingency table with two sets

of ordinal categories under possible model misspecification

on opinions about premarital sex (X) and teenage birth control (Y)

using the data by Agresti (2013, p.386)

[1]

[1] total frequency=926

[1] frequencies

	Y1	Y2	Y3	Y4
X1	81	68	60	38
X2	24	26	29	14
X3	18	41	74	42
X4	36	57	161	157

[1] percentages

	Y1	Y2	Y3	Y4
X1	8.7	7.3	6.5	4.1
X2	2.6	2.8	3.1	1.5
X3	1.9	4.4	8.0	4.5
X4	3.9	6.2	17.4	17.0

[1] row marginal(%)

	X1	X2	X3	X4
	26.7	10.0	18.9	44.4

[1] column marginal(%)

	Y1	Y2	Y3	Y4
	17.2	20.7	35.0	27.1

[1]

[1] The K(the number of catogories) by K

[1] design matrix for the linear-by-linear and saturated models

[1] (the 7th column is null, which will be filled by the

[1] linear-by-linear model)

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]
[1,]	1	0	0	1	0	0	0	1	0	0	0	0	0
[2,]	1	0	0	0	1	0	0	0	1	0	0	0	0
[3,]	1	0	0	0	0	1	0	0	0	1	0	0	0
[4,]	1	0	0	0	0	0	0	0	0	0	0	0	0
[5,]	0	1	0	1	0	0	0	0	0	0	1	0	0
[6,]	0	1	0	0	1	0	0	0	0	0	0	1	0

```

[7,] 0 1 0 0 0 1 0 0 0 0 0 0 0 1
[8,] 0 1 0 0 0 0 0 0 0 0 0 0 0 0
[9,] 0 0 1 1 0 0 0 0 0 0 0 0 0 0
[10,] 0 0 1 0 1 0 0 0 0 0 0 0 0 0
[11,] 0 0 1 0 0 1 0 0 0 0 0 0 0 0
[12,] 0 0 1 0 0 0 0 0 0 0 0 0 0 0
[13,] 0 0 0 1 0 0 0 0 0 0 0 0 0 0
[14,] 0 0 0 0 1 0 0 0 0 0 0 0 0 0
[15,] 0 0 0 0 0 1 0 0 0 0 0 0 0 0
[16,] 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

```

[,14] [,15] [,16]

```

```

[1,] 0 0 0
[2,] 0 0 0
[3,] 0 0 0
[4,] 0 0 0
[5,] 0 0 0
[6,] 0 0 0
[7,] 0 0 0
[8,] 0 0 0
[9,] 1 0 0
[10,] 0 1 0
[11,] 0 0 1
[12,] 0 0 0
[13,] 0 0 0
[14,] 0 0 0
[15,] 0 0 0
[16,] 0 0 0

```

```

[1]

```

[1] p-use=the pattern of parameter use (1:use,0:not use)

```

p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6

```

```

Model 1 (X,Y)          1 1 1 1 1 1
Model 2A (X,Y,linA)   1 1 1 1 1 1
Model 2B (X,Y,linB)   1 1 1 1 1 1
Model 3 (XY)(saturated) 1 1 1 1 1 1

```

```

p-use 7 p-use 8 p-use 9 p-use 10 p-use 11 p-use 12

```

```

Model 1 (X,Y)          0 0 0 0 0 0
Model 2A (X,Y,linA)   1 0 0 0 0 0
Model 2B (X,Y,linB)   1 0 0 0 0 0
Model 3 (XY)(saturated) 0 1 1 1 1 1

```

```

p-use 13 p-use 14 p-use 15 p-use 16

```

```

Model 1 (X,Y)          0 0 0 0
Model 2A (X,Y,linA)   0 0 0 0
Model 2B (X,Y,linB)   0 0 0 0
Model 3 (XY)(saturated) 1 1 1 1

```

```

[1]

```

[1] unstandardized u for Model (X,Y,linA)

```

[1] 1 2 3 4

```

[1] unstandardized v for Model (X,Y,linA)

```

[1] 1 2 3 4

```

[1] unstandardized u*v' for Model (X,Y,linA)

```

      [,1] [,2] [,3] [,4]
[1,]  1   2   3   4
[2,]  2   4   6   8
[3,]  3   6   9  12
[4,]  4   8  12  16
[1] standardized u for Model (X,Y,linA)
[1] -1.442 -0.645  0.151  0.948
[1] standardized v for Model (X,Y,linA)
[1] -1.650 -0.691  0.268  1.227
[1] standardized u*v' for Model (X,Y,linA)
      [,1] [,2] [,3] [,4]
[1,]  2.380  0.996 -0.387 -1.770
[2,]  1.065  0.446 -0.173 -0.792
[3,] -0.250 -0.105  0.041  0.186
[4,] -1.565 -0.655  0.254  1.164
[1]
[1] unstandadized u for Model (X,Y,linB)
[1] 1 2 4 5
[1] unstandardized v for Model (X,Y,linB)
[1] 1 2 4 5
[1] unstandardized u*v' for Model (X,Y,linB)
      [,1] [,2] [,3] [,4]
[1,]  1   2   4   5
[2,]  2   4   8  10
[3,]  4   8  16  20
[4,]  5  10  20  25
[1] standardized u for Model (X,Y,linB)
[1] -1.426 -0.842  0.325  0.909
[1] standardized v for Model (X,Y,linB)
[1] -1.574 -0.902  0.443  1.115
[1] standardized u*v' for Model (X,Y,linB)
      [,1] [,2] [,3] [,4]
[1,]  2.244  1.286 -0.631 -1.590
[2,]  1.326  0.759 -0.373 -0.939
[3,] -0.512 -0.293  0.144  0.363
[4,] -1.431 -0.820  0.403  1.014
[1]
[1] =====
[1] Model 1 (X,Y)
[1] "number of parameters=6"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1  p-use 2  p-use 3  p-use 4  p-use 5  p-use 6  p-use 7  p-use 8
      1      1      1      1      1      1      0      0
p-use 9  p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
      0      0      0      0      0      0      0      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6]
[1,]  1   0   0   1   0   0
[2,]  1   0   0   0   1   0
[3,]  1   0   0   0   0   1

```

```

[4,] 1 0 0 0 0 0
[5,] 0 1 0 1 0 0
[6,] 0 1 0 0 1 0
[7,] 0 1 0 0 0 1
[8,] 0 1 0 0 0 0
[9,] 0 0 1 1 0 0
[10,] 0 0 1 0 1 0
[11,] 0 0 1 0 0 1
[12,] 0 0 1 0 0 0
[13,] 0 0 0 1 0 0
[14,] 0 0 0 0 1 0
[15,] 0 0 0 0 0 1
[16,] 0 0 0 0 0 0
[1] .....
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] -0.5092049 -1.4859937 -0.8538072 -0.4565487 -0.2679576 0.2552906
[1] information-based ASEs
[1] 0.08050884 0.11482934 0.09026281 0.10135764 0.09587699 0.08408617
[1] robust ASEs(RASEs)
[1] 0.08050884 0.11482934 0.09026281 0.10135764 0.09587699 0.08408617
[1] RASE/ASE
[1] 1 1 1 1 1 1
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] -0.5900943 -1.5132182 -0.8172789 -0.5905505 -0.2549424 0.3108265
[1] information-based ASEs
[1] 0.08192023 0.11515865 0.08838669 0.10577443 0.09558068 0.08313713
[1] robust ASEs(RASEs)
[1] 0.09514767 0.12698725 0.09383876 0.12440861 0.10599820 0.08884008
[1] RASE/ASE
[1] 1.161 1.103 1.062 1.176 1.109 1.069
[1] .....
[1] estimation method=3
[1] parameter estimates
[1] -0.7159232 -1.5999395 -0.7950218 -0.7619474 -0.3011114 0.3526301
[1] information-based ASEs
[1] 0.08394540 0.11735047 0.08623481 0.11098265 0.09600212 0.08169733
[1] robust ASEs(RASEs)
[1] 0.1206917 0.1567767 0.0986794 0.1645183 0.1291253 0.0974380
[1] RASE/ASE
[1] 1.438 1.336 1.144 1.482 1.345 1.193
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] -0.5425720 -1.4928240 -0.8356766 -0.5161433 -0.2564579 0.2832485
[1] information-based ASEs

```

```

[1] 0.08111389 0.11480501 0.08942373 0.10336824 0.09567980 0.08367823
[1] robust ASEs(RASEs)
[1] 0.08625639 0.11879942 0.09158328 0.10933041 0.09964655 0.08552722
[1] RASE/ASE
[1] 1.063 1.035 1.024 1.058 1.041 1.022
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] -0.4875521 -1.4873785 -0.8688151 -0.4101978 -0.2826759 0.2297618
[1] information-based ASEs
[1] 0.08008370 0.11505980 0.09087097 0.09975734 0.09610047 0.08440270
[1] robust ASEs(RASEs)
[1] 0.07682780 0.11382946 0.09007916 0.09843038 0.09411921 0.08439530
[1] RASE/ASE
[1] 0.959 0.989 0.991 0.987 0.979 1.000
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] -0.4823368 -1.4888431 -0.8729089 -0.3972004 -0.2874998 0.2220574
[1] information-based ASEs
[1] 0.07997520 0.11515828 0.09102568 0.09930713 0.09617257 0.08449414
[1] robust ASEs(RASEs)
[1] 0.07594985 0.11393049 0.09023049 0.09817771 0.09386881 0.08476143
[1] RASE/ASE
[1] 0.950 0.989 0.991 0.989 0.976 1.003
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] -0.4741725 -1.4925053 -0.8797197 -0.3743045 -0.2964883 0.2079566
[1] information-based ASEs
[1] 0.07979862 0.11536623 0.09127306 0.09851673 0.09630696 0.08466085
[1] robust ASEs(RASEs)
[1] 0.07465291 0.11454079 0.09077221 0.09836340 0.09374491 0.08571429
[1] RASE/ASE
[1] 0.936 0.993 0.995 0.998 0.973 1.012
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] -0.4609923 -1.5040889 -0.8909048 -0.3252497 -0.3167800 0.1751952
[1] information-based ASEs
[1] 0.07949446 0.11594063 0.09165647 0.09684356 0.09660616 0.08505676
[1] robust ASEs(RASEs)
[1] 0.07390511 0.11795395 0.09360268 0.10153685 0.09544380 0.08904868
[1] RASE/ASE
[1] 0.930 1.017 1.021 1.048 0.988 1.047
[1]
[1] =====
[1] Model 2A (X,Y,linA)
[1] "number of parameters=7"
[1] the pattern of parameter use (1;use,0:not use)

```

```

p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7 p-use 8
      1      1      1      1      1      1      1      0
p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
      0      0      0      0      0      0      0      0

```

[1] the (reduced) design matrix for the model

```

      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    1    0    0    1    0    0  2.380
[2,]    1    0    0    0    1    0  0.996
[3,]    1    0    0    0    0    1 -0.387
[4,]    1    0    0    0    0    0 -1.770
[5,]    0    1    0    1    0    0  1.065
[6,]    0    1    0    0    1    0  0.446
[7,]    0    1    0    0    0    1 -0.173
[8,]    0    1    0    0    0    0 -0.792
[9,]    0    0    1    1    0    0 -0.250
[10,]   0    0    1    0    1    0 -0.105
[11,]   0    0    1    0    0    1  0.041
[12,]   0    0    1    0    0    0  0.186
[13,]   0    0    0    1    0    0 -1.565
[14,]   0    0    0    0    1    0 -0.655
[15,]   0    0    0    0    0    1  0.254
[16,]   0    0    0    0    0    0  1.164

```

[1]

[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))

[1]

[1] estimation method=1

[1] parameter estimates

[1] -0.5789915 -1.4473969 -0.7938966 -0.5298733 -0.1907992 0.3519626

[7] 0.3740116

[1] information-based ASEs

[1] 0.08657981 0.11664395 0.09141289 0.10810097 0.09961080 0.08660537

[7] 0.03694896

[1] robust ASEs(RASEs)

[1] 0.08737509 0.11595021 0.09150934 0.10813442 0.10030711 0.08794389

[7] 0.03852044

[1] RASE/ASE

[1] 1.009 0.994 1.001 1.000 1.007 1.015 1.043

[1]

[1] estimation method=2

[1] parameter estimates

[1] -0.5887809 -1.4539706 -0.8026313 -0.5296210 -0.1821819 0.3617084

[7] 0.3819869

[1] information-based ASEs

[1] 0.08681603 0.11680542 0.09159032 0.10844797 0.09982603 0.08674519

[7] 0.03715562

[1] robust ASEs(RASEs)

[1] 0.08872597 0.11717539 0.09275217 0.10968989 0.10194202 0.08939896

[7] 0.03955495

[1] RASE/ASE

[1] 1.022 1.003 1.013 1.011 1.021 1.031 1.065


```

[1] .....
[1] estimation method=3
[1] parameter estimates
[1] -0.5994112 -1.4586835 -0.8098657 -0.5299413 -0.1725591 0.3729617
[7] 0.3912852
[1] information-based ASEs
[1] 0.08711539 0.11692333 0.09175562 0.10886197 0.10007788 0.08690411
[7] 0.03741070
[1] robust ASEs(RASEs)
[1] 0.09208819 0.11969709 0.09512518 0.11389433 0.10544635 0.09237493
[7] 0.04195529
[1] RASE/ASE
[1] 1.057 1.024 1.037 1.046 1.054 1.063 1.121
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] -0.5837653 -1.4508834 -0.7983998 -0.5296878 -0.1865967 0.3566578
[7] 0.3778397
[1] information-based ASEs
[1] 0.08669044 0.11672972 0.09150237 0.10826685 0.09971376 0.08667312
[7] 0.03704671
[1] robust ASEs(RASEs)
[1] 0.08782081 0.11636855 0.09197630 0.10861013 0.10091251 0.08851810
[7] 0.03890969
[1] RASE/ASE
[1] 1.013 0.997 1.005 1.003 1.012 1.021 1.050
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] -0.5744749 -1.4435798 -0.7892146 -0.5301674 -0.1948251 0.3475958
[7] 0.3704864
[1] information-based ASEs
[1] 0.08648300 0.11654937 0.09132323 0.10794907 0.09951697 0.08654143
[7] 0.03686142
[1] robust ASEs(RASEs)
[1] 0.08731812 0.11595989 0.09136150 0.10816401 0.10007626 0.08761312
[7] 0.03831483
[1] RASE/ASE
[1] 1.010 0.995 1.000 1.002 1.006 1.012 1.039
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] -0.5730277 -1.4422458 -0.7876287 -0.5302888 -0.1961331 0.3462087
[7] 0.3693756
[1] information-based ASEs
[1] 0.08645358 0.11651613 0.09129352 0.10790135 0.09948765 0.08652088
[7] 0.03683434
[1] robust ASEs(RASEs)
[1] 0.08737255 0.11606124 0.09138258 0.10826709 0.10007422 0.08754811
[7] 0.03827667

```

```

[1] RASE/ASE
[1] 1.011 0.996 1.001 1.003 1.006 1.012 1.039
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] -0.5702210 -1.4394996 -0.7844361 -0.5305663 -0.1987036 0.3435314
[7] 0.3672464
[1] information-based ASEs
[1] 0.08639873 0.11644739 0.09123456 0.10781007 0.09943186 0.08648083
[7] 0.03678314
[1] robust ASEs(RASEs)
[1] 0.08757530 0.11641047 0.09152693 0.10858823 0.10017242 0.08747703
[7] 0.03823487
[1] RASE/ASE
[1] 1.014 1.000 1.003 1.007 1.007 1.012 1.039
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] -0.5624890 -1.4308006 -0.7748655 -0.5316748 -0.2060950 0.3362209
[7] 0.3615483
[1] information-based ASEs
[1] 0.08626263 0.11622710 0.09106401 0.10756731 0.09928578 0.08636802
[7] 0.03665094
[1] robust ASEs(RASEs)
[1] 0.08876222 0.11855830 0.09270210 0.11019863 0.10117436 0.08765867
[7] 0.03828761
[1] RASE/ASE
[1] 1.029 1.020 1.018 1.024 1.019 1.015 1.045
[1]
[1] =====
[1] Model 2B (X,Y,linB)
[1] "number of parameters=7"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7 p-use 8
      1      1      1      1      1      1      1      0
p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
      0      0      0      0      0      0      0      0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    1    0    0    1    0    0 2.244
[2,]    1    0    0    0    1    0 1.286
[3,]    1    0    0    0    0    1 -0.631
[4,]    1    0    0    0    0    0 -1.590
[5,]    0    1    0    1    0    0 1.326
[6,]    0    1    0    0    1    0 0.759
[7,]    0    1    0    0    0    1 -0.373
[8,]    0    1    0    0    0    0 -0.939
[9,]    0    0    1    1    0    0 -0.512
[10,]   0    0    1    0    1    0 -0.293
[11,]   0    0    1    0    0    1 0.144

```

```

[12,]    0    0    1    0    0    0  0.363
[13,]    0    0    0    1    0    0 -1.431
[14,]    0    0    0    0    1    0 -0.820
[15,]    0    0    0    0    0    1  0.403
[16,]    0    0    0    0    0    0  1.014

[1] .....
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] -0.5795756 -1.4688187 -0.8042863 -0.5286763 -0.2282483  0.3255431
[7]  0.3721345
[1] information-based ASEs
[1] 0.08650037 0.11687094 0.09095776 0.10743179 0.09923428 0.08534599
[7] 0.03587000
[1] robust ASEs(RASEs)
[1] 0.08703396 0.11638053 0.09070466 0.10673898 0.09977986 0.08565320
[7] 0.03643739
[1] RASE/ASE
[1] 1.006 0.996 0.997 0.994 1.005 1.004 1.016
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] -0.5838589 -1.4710607 -0.8195749 -0.5241862 -0.2262246  0.3330449
[7]  0.3745184
[1] information-based ASEs
[1] 0.08649825 0.11676808 0.09128244 0.10752307 0.09941259 0.08538768
[7] 0.03589420
[1] robust ASEs(RASEs)
[1] 0.08755863 0.11679762 0.09210622 0.10755621 0.10100091 0.08642853
[7] 0.03673744
[1] RASE/ASE
[1] 1.012 1.000 1.009 1.000 1.016 1.012 1.023
[1] .....
[1] estimation method=3
[1] parameter estimates
[1] -0.5881048 -1.4729835 -0.8345630 -0.5198470 -0.2252611  0.3393236
[7]  0.3769678
[1] information-based ASEs
[1] 0.08650388 0.11666130 0.09160863 0.10758826 0.09958929 0.08542453
[7] 0.03591541
[1] robust ASEs(RASEs)
[1] 0.08877127 0.11775366 0.09504082 0.10952173 0.10330792 0.08781803
[7] 0.03736947
[1] RASE/ASE
[1] 1.026 1.009 1.037 1.018 1.037 1.028 1.040
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] -0.5817118 -1.4699843 -0.8119540 -0.5264203 -0.2271009  0.3294318

```

```

[7] 0.3733124
[1] information-based ASEs
[1] 0.08649814 0.11682038 0.09111960 0.10748054 0.09932315 0.08536741
[7] 0.03588236
[1] robust ASEs(RASEs)
[1] 0.08721281 0.11651657 0.09121055 0.10700095 0.10025558 0.08596129
[7] 0.03654874
[1] RASE/ASE
[1] 1.008 0.997 1.001 0.996 1.009 1.007 1.019
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] -0.5774702 -1.4675597 -0.7966044 -0.5309379 -0.2296645 0.3214168
[7] 0.3709943
[1] information-based ASEs
[1] 0.08650519 0.11691909 0.09079755 0.10737731 0.09914691 0.08532352
[7] 0.03585734
[1] robust ASEs(RASEs)
[1] 0.08700783 0.11639425 0.09058094 0.10675852 0.09956373 0.08551005
[7] 0.03639430
[1] RASE/ASE
[1] 1.006 0.996 0.998 0.994 1.004 1.002 1.015
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] -0.5767784 -1.4671190 -0.7940468 -0.5316902 -0.2301947 0.3199959
[7] 0.3706241
[1] information-based ASEs
[1] 0.08650740 0.11693450 0.09074462 0.10735798 0.09911826 0.08531581
[7] 0.03585309
[1] robust ASEs(RASEs)
[1] 0.08702981 0.11643214 0.09062139 0.10682358 0.09954691 0.08549960
[7] 0.03639331
[1] RASE/ASE
[1] 1.006 0.996 0.999 0.995 1.004 1.002 1.015
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] -0.5754138 -1.4662068 -0.7889445 -0.5331889 -0.2313394 0.3170961
[7] 0.3698998
[1] information-based ASEs
[1] 0.08651274 0.11696425 0.09063964 0.10731771 0.09906187 0.08530010
[7] 0.03584459
[1] robust ASEs(RASEs)
[1] 0.08711505 0.11655697 0.09081833 0.10703437 0.09959224 0.08553449
[7] 0.03640892
[1] RASE/ASE
[1] 1.007 0.997 1.002 0.997 1.005 1.003 1.016
[1] .....
[1] estimation method=8

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[1] parameter estimates
[1] -0.5715111 -1.4632411 -0.7738462 -0.5375967 -0.2353878 0.3080624
[7] 0.3678682
[1] information-based ASEs
[1] 0.08653597 0.11704401 0.09033343 0.10718616 0.09890161 0.08525108
[7] 0.03581951
[1] robust ASEs(RASEs)
[1] 0.08763916 0.11729764 0.09222437 0.10820276 0.10030422 0.08606547
[7] 0.03656742
[1] RASE/ASE
[1] 1.013 1.002 1.021 1.009 1.014 1.010 1.021
[1]
[1] =====
[1] Model 3 (XY)(saturated)
[1] "number of parameters=15"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7 p-use 8
      1      1      1      1      1      1      0      1
p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
      1      1      1      1      1      1      1      1
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
[1,] 1 0 0 1 0 0 1 0 0 0 0 0 0
[2,] 1 0 0 0 1 0 0 1 0 0 0 0 0
[3,] 1 0 0 0 0 1 0 0 1 0 0 0 0
[4,] 1 0 0 0 0 0 0 0 0 0 0 0 0
[5,] 0 1 0 1 0 0 0 0 0 1 0 0 0
[6,] 0 1 0 0 1 0 0 0 0 0 1 0 0
[7,] 0 1 0 0 0 1 0 0 0 0 0 1 0
[8,] 0 1 0 0 0 0 0 0 0 0 0 0 0
[9,] 0 0 1 1 0 0 0 0 0 0 0 0 1
[10,] 0 0 1 0 1 0 0 0 0 0 0 0 0
[11,] 0 0 1 0 0 1 0 0 0 0 0 0 0
[12,] 0 0 1 0 0 0 0 0 0 0 0 0 0
[13,] 0 0 0 1 0 0 0 0 0 0 0 0 0
[14,] 0 0 0 0 1 0 0 0 0 0 0 0 0
[15,] 0 0 0 0 0 1 0 0 0 0 0 0 0
[16,] 0 0 0 0 0 0 0 0 0 0 0 0 0
      [,14] [,15]
[1,] 0 0
[2,] 0 0
[3,] 0 0
[4,] 0 0
[5,] 0 0
[6,] 0 0
[7,] 0 0
[8,] 0 0
[9,] 0 0
[10,] 1 0
[11,] 0 1

```

```

[12,]    0    0
[13,]    0    0
[14,]    0    0
[15,]    0    0
[16,]    0    0
[1] .....
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] .....
[1] estimation method=1
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
[7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13] 0.62542901  0.98909699  0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=2
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
[7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13] 0.62542901  0.98909699  0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=3
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
[7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13] 0.62542901  0.98909699  0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)

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```

[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=4
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454 0.02515856
[7] 2.22958986 1.59511608 0.43159984 2.01172337 1.63223375 0.70307994
[13] 0.62542901 0.98909699 0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=5
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454 0.02515856
[7] 2.22958986 1.59511608 0.43159984 2.01172337 1.63223375 0.70307994
[13] 0.62542901 0.98909699 0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=6
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454 0.02515856
[7] 2.22958986 1.59511608 0.43159984 2.01172337 1.63223375 0.70307994
[13] 0.62542901 0.98909699 0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387

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```

[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=7
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454 0.02515856
[7] 2.22958986 1.59511608 0.43159984 2.01172337 1.63223375 0.70307994
[13] 0.62542901 0.98909699 0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] .....
[1] estimation method=8
[1] parameter estimates
[1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454 0.02515856
[7] 2.22958986 1.59511608 0.43159984 2.01172337 1.63223375 0.70307994
[13] 0.62542901 0.98909699 0.54123692
[1] information-based ASEs
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
[1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
[8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
[1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
user system elapsed
7.14 0.03 7.17
[1] Thu Dec 06 10:38:56 2018

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