ROSEF Version 2.0 User's guide

A subroutine library for the **RO**tated solutions
with their asymptotic **S**tandard **E**rrors in **F**actor analysis

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Contents

1. General notes

(1) The subroutine library ROSEF contains Fortran90 subroutines for various rotated solutions with their asymptotic standard errors in factor analysis. Some of the rotated solutions and their asymptotic standard errors in principal component analysis are also available in ROSEF. The input data may be sample covariance/correlation matrices for observed variables, or factor loading matrices. (Admissible types of input data for a subroutine are described in each section for subroutines.) In all cases, multivariate normality is assumed for observed variables. When unrotated or rotated solutions are used as input data, they should be the solutions obtained by the method of maximum likelihood.

For the factor analysis model and associated algorithms, see e.g., Lawley and Maxwell (1971), Harman (1976), Shiba (1979) and Yanai, Shigemasu, Mayekawa and Ichikawa (1990). For the standard errors of rotated solutions, see e.g., Cudeck and O'Dell (1994).

(2) The standard errors of the estimates of loadings and factor/component correlations for standardized observed variables are independent of the standard errors of the estimates of scales (standard deviations of observed variables). Hence, the scales are set at unities without loss of generality and the corresponding equal standard errors for the unit scales are also given by the subroutines for standardized variables.

(3) The maximum problem sizes depend on machines and computer systems for users. The dimension sizes in the subroutines are specified by the arguments IR, IC and IH. The values of IR and IC should be greater than or equal to the number of observed variables (M) and that of common factors or principal components (MF), respectively. The value of IH should be greater than or equal to the row or column size of an augmented information matrix, which depends on rotation methods. That is, in factor analysis,

for the just identified confirmatory solution,

    IH>=M+M*MF-(MF**2-MF)/2,

for the orthomax and orthogonal procrustes rotations,

    IH>=M+M*MF+(MF**2-MF)/2,

for the oblique procrustes rotation and direct oblique rotation with the general symmetric family of quartic criteria,

    IH>=M+M*MF+(3/2)(MF**2-MF),

for the promax rotation

    IH>=3*M*MF+M+(1/2)*(3*MF**2+MF)

and for the orthoblique rotation

IH>=3*M*MF+M+(1/2)*(5*MF**2+3*MF).

On the other hand, in principal component analysis,
for the unrotated and orthomax solutions,

IH>=M*MF+(MF**2-MF)/2

and for the direct oblique rotation with the general symmetric family of quartic criteria,

IH>=M*MF+(3/2)(MF**2-MF).

(4) The subroutine INV which computes the inverse of a matrix is used frequently by other subroutines. It is recommended that INV is always attached to the other subroutines. Since the arguments in INV are called only by the other subroutines, users do not have to pay attention to the arguments in INV as long as users use INV in ROSEF. This holds for similar subroutines which are called indirectly by the subroutines which users use directly.

(5) The subroutines were tested by using DEC and Fujitsu Fortran90's and are expected to be compatible to most of different Fortran90's. Even in case when you have syntax errors, probably you can adapt the subroutines to your environment by slightly modifying them.

(6) ROSEF2.0 consists of 44 program files and two manual files. The 16 files beginning with "ro" are example programs, which take forms of UNIX shell script and should be changed to fit for users' environments, if necessary.

(7) Users can freely use the subroutines in ROSEF for research or educational purposes. However, when you publish an article using part or whole of ROSEF, you should refer to this user's guide.

(8) The author of ROSEF has no responsibility for any results directly or indirectly caused by using ROSEF.

* New features in ROSEF Version 2.0

The subroutines in principal component analysis have been revised by using the concise formulas for the standard errors of the solutions in principal component analysis given by Ogasawara (2002). Among the subroutines, those for unstandardized variables have been further simplified using the matrix expressions for the formulas (see e.g., Ogasawara, 2003). By these revisions, the amount of computation has been reduced to a large extent as well as the values of IH (see Section 1(3)). The revised subroutines are 3.2 UNSEUNP2A, 3.3 UNSESTP2, 4.5 OTSEUNP2A, 4.6 OTSESTP2, 6.5 QSEUNP2A, and 6.6

QSESTP2.

2. Common arguments

In the subroutines, the following common arguments are used. They are unchanged after a CALL statement is run in the main routine. (Note that some of other arguments take different values after a CALL statement is performed.) When arguments are arrays, their dimension sizes are specified by IR, IC and IH, which are also dummy arguments and were explained in the previous section. In example programs, most of the actual arguments in CALL statements take the same names as those of the corresponding dummy arguments but can take different names if necessary. The variables or arrays beginning with the letter I, J, K, L, M or N are integers, and others are real numbers with double precision.

M: The number of observed variables (input).
MF: The number of common factors or principal components to be rotated or kept unrotated (input).
N: The number of observations (input).
PSI(IR) or PS(IR): The vector of uniquenesses, where an uniqueness is defined as the variance of an unique factor, not its proportion in the total variance of the observed variable (input). For the usage of PSI(IR) in FAOC, see Section 3.
W: The orthomax weight which usually takes values in the range [0, M] (input).
INRM: INRM=1, when Kaiser's (row) normalization is employed in the orthomax rotation; INRM=0, when the normalization is not used in the orthomax rotation (input).
KAISER: KAISER=1, when Kaiser's (row) normalization is employed in the direct oblique rotation; KAISER=0, when the normalization is not used in the direct oblique rotation (input).
AK1, AK2, AK3, AK4: Weights for the general symmetric family of criteria in the direct oblique rotation (see 6.1 (3); input).
ICOR: When ICOR=1, the asymptotic correlations between the parameter estimates of interest are printed; when ICOR=0, the output of the correlation matrix, which tends to be voluminous, is suppressed (input).

3. Unrotated solutions

3.1 FAOC: Unrotated solution in factor analysis for unstandardized variables

SUBROUTINE FAOC (M, MF, N, S, P, D, ND, PSI, IR, IC, IH)

(1) Purpose

FAOC derives an unrotated solution (loadings and uniquenesses) with their asymptotic standard errors in an exploratory factor analysis model for unstandardized variables. The loading matrix takes a form of a just identified confirmatory orthogonal factor analysis model with (MF**2-MF)/2 fixed loadings being zero in appropriate locations.

(2) Arguments

For M, MF, N, IR, IC and IH, see Sections 1 and 2.

S(IR, IR): An unbiased sample covariance matrix (input).

P(IR, IR): The inverse of a fitted covariance matrix (output).

D(IR, IC): The initial value for the loading matrix (input).

The estimated loading matrix (output).

ND(IR, IC): The parameter pattern matrix, in which 0 denotes a fixed zero loading and 1 denotes a free parameter to be estimated (input). Note that the number of 0's should be (MF**2-MF)/2. An example of selecting appropriate locations of fixed zero loadings is that the upper-right triangular (MF**-MF)/2 elements in ND(,) are set equal to zero.

The 0's are not changed after a CALL statement is carried out, while 1's are replaced by the positive integers representing the order of parameters (output). Usually, users do not use the output of ND(,)

(3) Remarks

The model for standardized variables (i.e., factor analysis of correlation matrices) is not assumed. However, when a correlation matrix is input, the parameter estimates are equivalent to those for standardized variables. Note that in this case, the asymptotic standard errors can not be used since they are different from the exact ones for standardized variables.

Iterative computation is used in FAOC. The criterion of convergence (EPS) need not be changed for usual problems, but can be modified. EPS is presently 0.000001. When none of the absolute values of the elements of the gradient vector of the fit function is greater than the value of EPS, the iteration is stopped. The iteration is also stopped when the number of iterations becomes more than 1,000, which is sufficient for usual problems. The maximum number is specified by IMAX and can be changed if necessary.

When convergence is not obtained, the initial values should be changed to more appropriate values. However, it is not guaranteed that convergence is obtained with revised initial values when the factor analysis model does not fit the data or when there exist some anomalies.

The subroutine INV is used in FAOC.

3.2 UNSEUNP2A: The asymptotic standard errors of the unrotated loadings in principal component analysis for unstandardized variables

SUBROUTINE UNSEUNP2A(P, M, MF, N, IR, IC, IH, ICOR)

(1) Purpose

UNSEUNP2A derives unrotated loadings with their asymptotic standard errors in principal component analysis for unstandardized variables. The vector of loadings for an unrotated component is defined as a normalized eigenvector of a covariance matrix for observed variables multiplied by the square root of the corresponding eigenvalue. That is, the sum of the squared elements of the loading vector becomes equal to the eigenvalue. (For the asymptotic standard errors, see Ogasawara, 2000c, 2002.)

(2) Arguments

For M, MF, N, IR, IC, IH and ICOR, see Sections 1 and 2.

P(IR, IR): An unbiased sample covariance matrix (input).

(3) Remarks

For the computation of the eigenvectors and eigenvalues of a symmetric matrix, Jacobi's method (see e.g., Shiba, 1979) is used in the subroutine EIGEN. The criterion of convergence (EPS) need not be changed for usual problems, but can be modified. EPS is presently 0.000001. When none of the absolute values of the off-diagonal elements in the diagonalizing process is greater than or equal to the value of EPS, the iteration is stopped. The iteration is also stopped when the number of the iterations becomes more than 500, which is sufficient for usual problems. The maximum number is specified by MAXI and can be changed if necessary.

The subroutines INV and EIGEN are used in UNSEUNP2A.

3.3 UNSESTP2: The asymptotic standard errors of the unrotated loadings in principal component analysis for standardized variables

SUBROUTINE UNSESTP2(P, M, MF, N, IR, IC, IH, ICOR)

(1) Purpose

UNSESTP2 derives unrotated loadings with their asymptotic standard errors in principal component analysis for standardized variables. The vector of loadings for an unrotated component is defined as a normalized eigenvector of a correlation matrix for observed variables multiplied by the square root of the corresponding eigenvalue. That is, the sum of the squared elements of the loading vector becomes equal to the eigenvalue. (For the asymptotic standard errors, see Ogasawara, 2000c, 2002.)

(2) Arguments

For M, MF, N, IR, IC, IH and ICOR, see Sections 1 and 2.

P(IR, IR): A sample correlation matrix for observed variables (input).

(3) Remarks

For the computation of the eigenvectors and eigenvalues of a correlation matrix, see 3.2 (3) Remarks. For the scales (the estimates of the standard deviations of observed variables) see Section 1(2).

The subroutines INV and EIGEN are used in UNSESTP2.

4. Orthomax rotation

4.1 OTMAX: Orthomax solution

SUBROUTINE OTMAX(M, MF, A, CT, W, CM, INRM, IR, IC)

(1) Purpose

OTMAX obtains the orthomax solution from an unrotated factor/component loading matrix.

(2) Arguments

For M, MF, W, INRM, IR and IC, see Sections 1 and 2.

A(IR, IC): An unrotated factor/component loading matrix (input).

The corresponding orthomax-rotated loading matrix (output).

CT(IC): The contributions of orthomax rotated factors/components (output).

CM(IR): The communalities of observed variables (output).

(3) Remarks

In OTMAX, iterative computation is used. The maximum number of iterations (IMAX) is set at 500. The number is sufficient for ordinary problems, but can be changed if necessary. The criterion of convergence is specified by EPS which is set at 0.000001 (radian). When none of the amount of rotation in each pair of common factors or principal components is greater than or equal to EPS, the iteration is stopped. EPS can be changed if necessary, though the above value is sufficient for usual problems.

For the orthomax solutions and their asymptotic standard errors, see Archer and Jennrich (1973), Jennrich (1974), Ogasawara (1996, 1998a, 2000a, 2002) and Hayashi and Yung (1999).

4.2 OTSEUN: The standard errors of the orthomax solution in factor analysis for unstandardized variables

SUBROUTINE OTSEUN(M, MF, N, D, U, S1, W, HS1, INRM, IR, IC, IH, ICOR)

(1) Purpose

OTSEUN outputs the asymptotic standard errors of the estimates of the orthomax-rotated factor loadings, the uniquenesses, the contributions and the communalities for unstandardized variables.

(2) Arguments

For M, MF, N, W, INRM, IR, IC, IH and ICOR see Sections 1 and 2.

D(IR, IC): The orthomax rotated loading matrix for unstandardized variables (input).

U(IR): The uniquenesses in unstandardized variables (input).

S1(IC): The contributions of the orthomax rotated factors (input).

HS1(IC): The communalities in unstandardized variables (output).

(3) Remarks

The subroutine INV is used in OTSEUN.

4.3 OTSEST: The standard errors of the orthomax solution in factor analysis for standardized variables

SUBROUTINE OTSEST(M, MF, N, D, S1, W, HS1, INRM, IR, IC, IH, ICOR)

(1) Purpose

OTSEUN outputs the asymptotic standard errors of the estimates of the scales (standard deviations of observed variables), the orthomax-rotated factor loadings, the contributions and the communalities for standardized variables.

(2) Arguments

For M, MF, N, W, INRM, IR, IC, IH and ICOR see Sections 1 and 2.

D(IR, IC): The orthomax-rotated loading matrix for standardized variables (input).

S1(IC): The contributions of the orthomax-rotated factors (input).

HS1(IC): The communalities in unstandardized variables (output).

(3) Remarks

For standardized variables, the sum of the communality and uniqueness for a standardized variable is always one. Hence, the asymptotic standard error of an uniqueness is equal to that of communality. For the scales, see Section 3.3 (3) Remarks. The subroutine INV is used in OTSEST.

4.4 Example: Usage for FAOC, OTMAX and OTSEUN

(1) Program

```
#!/bin/csh
cat > $1.f << EOF
C
C     STANDARD ERRORS OF ORTHOMAX SOLUTION FOR UNSTANDARDIZED VARIABLES
C
C     <ROTSE>                              '99.9.14-27 '99.10.7 2003.4.21
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*4 AN(20)
      DIMENSION S1(3),P(20,20),S(20,20),
     &D(20,3),ND(20,3),PSI(20),HS1(20)
      OPEN(5,FILE='$1.cards',STATUS='OLD')
  122 READ(5,123,END=99)(AN(I),I=1,20)
  123 FORMAT(20A4)
      WRITE(6,124)(AN(I),I=1,20)
  124 FORMAT(////,80('-'),/20A4)
      READ(5,125)W,INRM,M,MF,N
  125 FORMAT(F5.0,4I5)
      WRITE(6,126)N
  126 FORMAT(//,'N=',I4)
C
      DO 9 I=1,M
      READ(5,101)(S(I,J),J=1,I)
  101 FORMAT(12F5.0)
      DO 9 J=1,I
      S(J,I)=S(I,J)
    9 CONTINUE
C
      DO 63 I=1,MF
```

```
      READ(5,206)(ND(J,I),J=1,M)
  206 FORMAT(40I2)
   63 WRITE(6,118)(ND(J,I),J=1,M)
  118 FORMAT(/,'PATTERN OF INITIAL ML LOADINGS',12I3)
      DO 40 I=1,MF
      READ(5,117)(D(J,I),J=1,M)
   40 WRITE(6,119)(D(J,I),J=1,M)
  119 FORMAT(/,'INITIAL LOADINGS',12F5.2)
      READ(5,117)(PSI(I),I=1,M)
  117 FORMAT(12F4.1)
      WRITE(6,120)(PSI(I),I=1,M)
  120 FORMAT(/'INITIAL PSI',12F5.2)
C
      IR=20
      IC=3
      IH=150
      CALL FAOC(M,MF,N,S,P,D,ND,PSI,IR,IC,IH)
C
      CALL OTMAX(M,MF,D,S1,W,HS1,INRM,IR,IC)
C
      ICOR=1
      CALL OTSEUN(M,MF,N,D,PSI,S1,W,HS1,INRM,IR,IC,IH,ICOR)
      GO TO 122
   99 STOP
      END
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/faoc >> $1.f
cat ~/haru/otmax >> $1.f
cat ~/haru/otseun >> $1.f
cat > $1.cards << EOF
8 PHYSICAL VARIABLES (HARMAN,1976,P22)
  1.0    1    8    2  305
 1.
 .846 1.
 .805 .881 1.
```

```
 .859 .826 .801 1.
 .473 .376 .380 .436 1.
 .398 .326 .319 .329 .762 1.
 .301 .277 .237 .327 .730 .583 1.
 .382 .415 .345 .365 .629 .577 .539 1.
 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1
 .91 .94 .91 .89 .51 .43 .36 .46
 .00-.12-.11-.03 .81 .67 .67 .50
 .17 .11 .17 .20 .09 .36 .42 .54  ---- PSI
EOF
f90 -o prog$1 $1.f >& $1.u
date >> $1.u
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
hostname >> $1.u
```

(2) Output (part)

```
SES OF UNIQUENESSES AND ORTHOMAX LOADINGS

 FOR UNSTANDARDIZED VARIABLES

UNIQUENESSES
     1  ESTIMATE= 0.16976749D+00  SE= 0.17523013D-01  T= 0.96882588D+01
     2  ESTIMATE= 0.10706770D+00  SE= 0.14885775D-01  T= 0.71926182D+01
     3  ESTIMATE= 0.16616792D+00  SE= 0.18028776D-01  T= 0.92168167D+01
     4  ESTIMATE= 0.19941761D+00  SE= 0.19732344D-01  T= 0.10106129D+02
     5  ESTIMATE= 0.89118248D-01  SE= 0.29107919D-01  T= 0.30616496D+01
     6  ESTIMATE= 0.36370542D+00  SE= 0.36205815D-01  T= 0.10045497D+02
     7  ESTIMATE= 0.41634658D+00  SE= 0.39803408D-01  T= 0.10460074D+02
     8  ESTIMATE= 0.53673400D+00  SE= 0.46285388D-01  T= 0.11596187D+02

ORTHOMAX LOADINGS OF FACTOR 1
     9  ESTIMATE= 0.86284556D+00  SE= 0.41593755D-01  T= 0.20744594D+02
    10  ESTIMATE= 0.92620109D+00  SE= 0.41457648D-01  T= 0.22340898D+02
    11  ESTIMATE= 0.89415370D+00  SE= 0.42689418D-01  T= 0.20945558D+02
```

```
    12  ESTIMATE= 0.85686241D+00  SE= 0.42604976D-01  T= 0.20111792D+02
    13  ESTIMATE= 0.22681731D+00  SE= 0.34481896D-01  T= 0.65778666D+01
    14  ESTIMATE= 0.18905031D+00  SE= 0.39897954D-01  T= 0.47383458D+01
    15  ESTIMATE= 0.12889929D+00  SE= 0.40175305D-01  T= 0.32084210D+01
    16  ESTIMATE= 0.27344669D+00  SE= 0.46567684D-01  T= 0.58720269D+01

ORTHOMAX LOADINGS OF FACTOR 2
    17  ESTIMATE= 0.29279695D+00  SE= 0.36098515D-01  T= 0.81110524D+01
    18  ESTIMATE= 0.18730680D+00  SE= 0.34334767D-01  T= 0.54553101D+01
    19  ESTIMATE= 0.18525990D+00  SE= 0.35605111D-01  T= 0.52031828D+01
    20  ESTIMATE= 0.25762220D+00  SE= 0.36675132D-01  T= 0.70244382D+01
    21  ESTIMATE= 0.92705753D+00  SE= 0.43257172D-01  T= 0.21431302D+02
    22  ESTIMATE= 0.77495455D+00  SE= 0.47969286D-01  T= 0.16155224D+02
    23  ESTIMATE= 0.75301951D+00  SE= 0.49478718D-01  T= 0.15219059D+02
    24  ESTIMATE= 0.62329199D+00  SE= 0.50270717D-01  T= 0.12398709D+02
```

4.5 OTSEUNP2A: The standard errors of the orthomax solution in principal component analysis for unstandardized variables

SUBROUTINE OTSEUNP2A (P, M, MF, N, W, INRM, IR, IC, IH, ICOR)

(1) Purpose

OTSEUNP2A outputs the asymptotic standard errors of the estimates of the orthomax-rotated component loadings for unstandardized variables. (For the asymptotic standard errors, see Ogasawara, 2000c, 2002.)

(2) Arguments

For M, MF, N, W, INRM, IR, IC, IH and ICOR see Sections 1 and 2.

P(IR, IR): An unbiased sample covariance matrix for observed variables (input).

(3) Remarks

Note that the input data set is not the rotated loading matrix as is for OTSEUN2A, but is the covariance matrix for observed variables. The covariance matrix is necessary for the computation of the standard errors of the estimated component loadings as well as for the rotated solution. The subroutine INV, EIGEN and OTMAX are used in OTSEUNP2A. See also Section 3.2 (3) Remarks and Section 4.1.

4.6 OTSESTP2: The standard errors of the orthomax solution in principal component analysis for standardized variables

SUBROUTINE OTSESTP2(P, M, MF, N, W, INRM, IR, IC, IH, ICOR)

(1) Purpose

OTSESTP2 outputs the asymptotic standard errors of the estimates of the scales (standard deviations of observed variables) and the orthomax-rotated component loadings for standardized variables. (For the asymptotic standard errors, see Ogasawara, 2000c, 2002.)

(2) Arguments

For M, MF, N, W, INRM, IR, IC, IH and ICOR see Sections 1 and 2.

P(IR, IR): A sample correlation matrix for observed variables (input).

(3) Remarks

Note that the input data set is not the rotated loading matrix as is for OTSEST, but is the correlation matrix for observed variables. The correlation matrix is necessary for the computation of the standard errors of the estimated component loadings as well as for the rotated solution. The subroutine INV, EIGEN and OTMAX are used in OTSESTP2. For the scales (standard deviations of observed variables), see Section 1(2). See also Section 3.2 (3) Remarks and 4.1.

4.7 Example: Usage for UNSEUNP2A and OTSEUNP2A

(1) Program

```
#!/bin/csh
cat > $1.f <<EOF
C
C     A CONCISE METHOD FOR THE STANDARD ERRORS
C     OF THE UNROTATED AND ORTHOMAX-ROTATED SOLUTIONS
C     IN PCA FOR UNSTANDARDIZED VARIABLES
C
C     <ROTSEP2>                                        2000.2.21-2003.4.18
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*4 AN(20)
      DIMENSION S(20,20),R(20,20),D(20)
      OPEN(5,FILE='$1.cards')
```

```
    1 READ(5,102,END=99)(AN(I),I=1,20)
  102 FORMAT(20A4)
      WRITE(6,103)(AN(I),I=1,20)
  103 FORMAT(////,80('-')/20A4)
      READ(5,104)W,INRM,M,MF,N
  104 FORMAT(F5.0,4I5)
      WRITE(6,105)N
  105 FORMAT(//,'N=',I4/)
C
      DO 6 I=1,M
      READ(5,107)(S(I,J),J=1,I)
  107 FORMAT(12F5.0)
      WRITE(6,117)(S(I,J),J=1,I)
  117 FORMAT('COV=',8F8.4)
      D(I)=SQRT(S(I,I))
      DO 6 J=1,I
      S(J,I)=S(I,J)
      R(I,J)=S(I,J)/(D(I)*D(J))
    6 R(J,I)=R(I,J)
C
      WRITE(6,120)
  120 FORMAT(' ')
      WRITE(6,118)(D(I),I=1,M)
  118 FORMAT(' SD=',8F8.4)
      WRITE(6,120)
      DO 8 I=1,M
      WRITE(6,119)(R(I,J),J=1,I)
  119 FORMAT('COR=',8F8.4)
    8 CONTINUE
C
C     PCA
      IR=20
      IC=MF
      IH=M**2+(M**2-M)/2
      ICOR=1
C
```

```
      CALL UNSEUNP2A(S,M,MF,N,IR,IC,IH,ICOR)
C
      CALL OTSEUNP2A(S,M,MF,N,W,INRM,IR,IC,IH,ICOR)
C
      GO TO 1
   99 STOP
      END
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/eigen >> $1.f
cat ~/haru/otmax >> $1.f
cat ~/haru/unseunp2a >> $1.f
cat ~/haru/otseunp2a >> $1.f
cat > $1.cards << EOF
6 SCHOOL SUBJECTS (LAWLEY & MAXWELL,1971,P66)
  1.0    1    6    2  220
 1.
 .439 1.
 .410 .351 1.
 .288 .354 .164 1.
 .329 .320 .190 .595 1.
 .248 .329 .181 .470 .464 1.
EOF
f90 -o prog$1 $1.f >& $1.u
date >> $1.u
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
hostname >> $1.u
```

(2) Output (part)

```
SES OF ORTHOMAX-ROTATED LOADINGS FOR UNSTANDARDIZED VARIABLES

LOADINGS OF COMPONENT  1
  1  ESTIMATE= 0.22531099D+00  SE= 0.73448555D-01  T= 0.30676027D+01
  2  ESTIMATE= 0.34938651D+00  SE= 0.92393803D-01  T= 0.37814929D+01
  3  ESTIMATE=-0.25888359D-02  SE= 0.62765924D-01  T=-0.41245881D-01
```

```
  4  ESTIMATE= 0.83307503D+00  SE= 0.64280801D-01  T= 0.12959936D+02
  5  ESTIMATE= 0.81360421D+00  SE= 0.66207487D-01  T= 0.12288704D+02
  6  ESTIMATE= 0.74990372D+00  SE= 0.78680550D-01  T= 0.95309924D+01

LOADINGS OF COMPONENT  2
  7  ESTIMATE= 0.76394352D+00  SE= 0.79076330D-01  T= 0.96608368D+01
  8  ESTIMATE= 0.66043529D+00  SE= 0.92825411D-01  T= 0.71148114D+01
  9  ESTIMATE= 0.82089386D+00  SE= 0.81666098D-01  T= 0.10051832D+02
 10  ESTIMATE= 0.14727008D+00  SE= 0.62585096D-01  T= 0.23531174D+01
 11  ESTIMATE= 0.17989328D+00  SE= 0.64123260D-01  T= 0.28054295D+01
 12  ESTIMATE= 0.15423710D+00  SE= 0.79183708D-01  T= 0.19478389D+01
```

5. Procrustes rotation

5.1 PROCO: Orthogonal procrustes solution

SUBROUTINE PROCO(M, MF, D, IP, DP, S1, IOR, IR, IC)

(1) Purpose

PROCO obtains orthogonally rotated loadings by Browne's (1972a) orthogonal procrustes method with fully or partially specified targets.

(2) Arguments

For M, MF, IR and IC, see Sections 1 and 2.

D(IR, IC): An unrotated loading matrix of orthogonal factors (input).

The orthogonally procrustes-rotated loading matrix (output).

IP(IR, IC): A target pattern matrix. When a target for a loading is specified, the corresponding element of IP(,) is one, otherwise zero (input).

DP(IR, IC): A target matrix, where the elements corresponding to not specified targets can be arbitrary values (e.g., zero) (input).

S1(IC): The contributions of the orthogonally rotated factors (output).

IOR: The unrotated factors may be reordered and/or reflected to approximate targets as close as possible. When the reordering or reflection is performed, IOR=1; when the reordering or reflection is not carried out, IOR=0 (output).

(3) Remarks

The Newton-Raphson method is used. The convergence criterion (EPS=0.0000001) and the maximum number of iterations (IMAX=200) may be sufficient for usual problems. They can be changed if necessary.

5.2 PRCSEUN: The asymptotic standard errors of the orthogonal procrustes

solution in factor analysis for unstandardized variables

SUBROUTINE PRCSEUN(M, MF, N, D, PSI, IP, DP, S1, IR, IC, IH, ICOR)

(1) Purpose

PRCSEUN output the asymptotic standard errors of the orthogonal procrustes solution with fully or partially specified targets by Browne's (1972a) method in factor analysis for unstandardized variables.

(2) Arguments

For M, MF, N, PSI, IR, IC, IH and ICOR see Sections 1 and 2.

D(IR, IC): The orthogonally rotated loadings by Browne's (1972a) method for unstandardized variables (input).

IP(IR, IC): A target pattern matrix. When a target is specified for a loading the corresponding element of IP(,) is one, otherwise zero (input).

DP(IR, IC): A target matrix, where the elements corresponding to not specified targets can be arbitrary values (e.g., zero) (input).

S1(IC): The contributions of the orthogonally rotated factors (input).

(3) Remarks

Note that S1(IC) is for input while it was for output in PROCO. The subroutine INV is used in PRCSEUN.

5.3 PRCSEST: The asymptotic standard errors of the orthogonal procrustes solution in factor analysis for standardized variables

SUBROUTINE PRCSEST(M, MF, N, D, IP, DP, S1, IR, IC, IH, ICOR)

(1) Purpose

PRCSEST outputs the asymptotic standard errors of the estimates of the orthogonal procrustes solution with fully or partially specified targets by Browne's (1972a) method in factor analysis for standardized variables.

(2) Arguments

For M, MF, N, IR, IC, IH and ICOR see Sections 1 and 2.

D(IR, IC): The orthogonally rotated loadings by Browne's (1972a) method for standardized variables (input).

IP(IR, IC): A target pattern matrix. When a target is specified for a loading the corresponding element of IP(,) is one, otherwise zero (input).

DP(IR, IC): A target matrix, where the elements corresponding to not specified targets can be arbitrary values (e.g., zero) (input).

S1(IC): The contributions of the orthogonally rotated factors (input).

(3) Remarks

Note that S1(IC) is for input while it was for output in PROCO. The subroutine INV is used in PRCSEST. For the scales (standard deviations of observed variables), see Section 1(2).

5.4 PROCB: Oblique procrustes solution

SUBROUTINE PROCB(M, MF, D, IP, DP, C, IOR, IR, IC)

(1) Purpose

PROCB obtains the obliquely rotated loadings and factor correlations by Browne's (1972b) oblique procrustes method with fully or partially specified targets.

(2) Arguments

For M, MF, IR and IC, see Sections 1 and 2.

D(IR, IC): An unrotated loading matrix of orthogonal or oblique factors (input).

The obliquely procrustes-rotated loading matrix (output).

IP(IR, IC): A target pattern matrix. When a target for a loading is specified, the corresponding element of IP(,) is one, otherwise zero (input).

DP(IR, IC): A target matrix, where the elements corresponding to not specified targets can be arbitrary values (e.g., zero) (input).

C(IC, IC): The factor correlation matrix of unrotated factors, which is the identity matrix when the unrotated factors are orthogonal (input).

The factor correlation matrix of obliquely rotated factors by the oblique procrustes method (output).

IOR: The unrotated factors may be reordered and/or reflected to approximate targets as close as possible. When the reordering or reflection is performed, IOR=1; when the reordering or reflection is not carried out, IOR=0 (output).

(3) Remarks

A Newton-Raphson method is used. The convergence criterion (EPS=0.0000001) and the maximum number of iterations (IMAX=200) may be sufficient for usual problems. They can be changed if necessary.

5.5 PRCBSEUN: The asymptotic standard errors of the oblique procrustes solution in factor analysis for unstandardized variables

SUBROUTINE PRCBSEUN(M, MF, N, D, C, PS, IP, DP, IR, IC, IH, ICOR)

(1) Purpose

PRCBSEUN outputs the asymptotic standard errors of the oblique procrustes solution with fully or partially specified targets by Browne's (1972b) method in factor analysis for unstandardized variables.

(2) Arguments

For M, MF, N, PS, IR and IC, see Sections 1 and 2.

D(IR, IC): An obliquely rotated loadings by Browne's (1972b) method for unstandardized variables (input).

C(IC, IC): The factor correlation matrix for the obliquely rotated factors for unstandardized variables (input).

IP(IR, IC): A target pattern matrix. When a target for a loading is specified, the corresponding element of IP(,) is one, otherwise zero (input).

DP(IR, IC): A target matrix, where the elements corresponding to not specified targets can be arbitrary values (e.g., zero) (input).

(3) Remarks

The subroutine INV is used in PRCBSEUN.

5.6 PRCBSEST: The asymptotic standard errors of the oblique procrustes solution in factor analysis for standardized variables

SUBROUTINE PRCBSEST(M, MF, N, D, C, IP, DP, IR, IC, IH, ICOR)

(1) Purpose

PRCBSEST outputs the asymptotic standard errors of the oblique procrustes solution with fully or partially specified targets by Browne's (1972b) method in factor analysis for standardized variables.

(2) Arguments

For M, MF, N, IR and IC, see Sections 1 and 2.

D(IR, IC): An obliquely rotated loadings by Browne's (1972b) method for standardized variables (input).

C(IC, IC): The factor correlation matrix for the obliquely rotated factors for standardized variables (input).

IP(IR, IC): A target pattern matrix. When a target for a loading is specified, the corresponding element of IP(,) is one, otherwise zero (input).

DP(IR, IC): A target matrix, where the elements corresponding to not specified targets can be arbitrary values (e.g., zero) (input).

(3) Remarks

The subroutine INV is used in PRCBSEST. For the scales (standard deviations of observed variables), see Section 1(2).

5.7 Example: Usage for FAOC, PROCB and PRCBSEST

(1) Program

```
#!/bin/csh
cat > $1.f << EOF
C
C     STANDARD ERRORS OF THE  DIRECT-OBLIQUE PROCRUSTES SOLUTION (BROWNE,1972B)
C      FOR STANDARDIZED VARIABLES
C
C     <ROPRCBSER>                             '99.10.7  2003.4.21
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*4 AN(20)
      DIMENSION C(3,3),P(20,20),
     *D(20,3),ND(20,3),PS(20),CT(3),S(20,20),IP(20,3),DP(20,3)
      OPEN(5,FILE='$1.cards',STATUS='OLD')
   77 READ(5,120,END=99)(AN(I),I=1,20)
  120 FORMAT(20A4)
      WRITE(6,121)(AN(I),I=1,20)
  121 FORMAT(////,80('-')/20A4)
      READ(5,101)M,MF,N
  101 FORMAT(3I5)
      WRITE(6,102)N
  102 FORMAT(//'N=',I4)
      AM=M
      DO 9 I=1,M
      READ(5,123)(S(I,J),J=1,I)
  123 FORMAT(12F5.0)
      DO 9 J=1,I
      S(J,I)=S(I,J)
    9 CONTINUE
C
      DO 63 I=1,MF
      READ(5,127)(ND(J,I),J=1,M)
  127 FORMAT(40I2)
   63 WRITE(6,118)(ND(J,I),J=1,M)
```

```
  118 FORMAT(/,'PATTERN OF INITIAL ML LOADINGS=',12I3)
      DO 126 I=1,MF
      READ(5,117)(D(J,I),J=1,M)
  126 WRITE(6,119)(D(J,I),J=1,M)
  119 FORMAT(/,'INITIAL VALUE=',12F5.2)
  117 FORMAT(12F4.1)
      READ(5,117)(PS(I),I=1,M)
C
      IR=20
      IC=3
      IH=150
      CALL FAOC(M,MF,N,S,P,D,ND,PS,IR,IC,IH)
C
      DO 133 I=1,MF
      READ(5,128)(IP(J,I),J=1,M)
  128 FORMAT(20I2)
  133 CONTINUE
      DO 130 I=1,MF
      READ(5,132)(DP(J,I),J=1,M)
  132 FORMAT(20F4.1)
  130 CONTINUE
      DO 134 I=1,MF
      READ(5,132)(C(I,J),J=1,MF)
  134 WRITE(6,135)(C(I,J),J=1,MF)
  135 FORMAT(/'FAC.COR.=',6F6.3)
C
      CALL PROCB(M,MF,D,IP,DP,C,IOR,IR,IC)
C
      WRITE(6,125)IOR
  125 FORMAT(/'IOR(1:REORDERED OR REFLECTED, 0:OTHERWISE)=',I2)
      ICOR=1
C
      CALL PRCBSEST(M,MF,N,D,C,IP,DP,IR,IC,IH,ICOR)
C
      GO TO 77
   99 STOP
```

```
      END
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/faoc >> $1.f
cat ~/haru/procb >> $1.f
cat ~/haru/prcbsest >> $1.f
cat > $1.cards << EOF
8 PHYSICAL VARIABLES (HARMAN,1976,P22)
    8    2  305
 1.
 .846 1.
 .805 .881 1.
 .859 .826 .801 1.
 .473 .376 .380 .436 1.
 .398 .326 .319 .329 .762 1.
 .301 .277 .237 .327 .730 .583 1.
 .382 .415 .345 .365 .629 .577 .539 1.
 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1
 .91 .94 .91 .89 .51 .43 .36 .46
 .00-.12-.11-.03 .81 .67 .67 .50
 .17 .11 .17 .20 .09 .36 .42 .54  ---- PSI
 0 0 0 0 1 1 1 1
 1 1 1 1 0 0 0 0 -- PATTERN FOR PROCRUSTES
   0   0   0   0  .0  .0  .0  .0
  .0  .0  .0  .0   0   0   0   0 -- TARGET
 1.0 0.0
 0.0 1.0
EOF
f90 -o prog$1 $1.f >& $1.u
date >> $1.u
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
hostname >> $1.u
```

(2) Output

```
SES FOR SCALES, FACTOR COR. AND LOADINGS

SCALES (SET AT ONES WITHOUT LOSS OF GENERALITY)
  1 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  2 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  3 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  4 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  5 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  6 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  7 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
  8 ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02

FAC.COR.:(2,1),(3,1),(3,2),...
  9 ESTIMATE= 0.47842310D+00  SE= 0.48078547D-01  T= 0.99508644D+01

PROCRUSTES LOADINGS OF FACTOR 1
 10 ESTIMATE= 0.87018622D+00  SE= 0.18506932D-01  T= 0.47019474D+02
 11 ESTIMATE= 0.96970249D+00  SE= 0.13845718D-01  T= 0.70036273D+02
 12 ESTIMATE= 0.93490611D+00  SE= 0.16421770D-01  T= 0.56930898D+02
 13 ESTIMATE= 0.87344952D+00  SE= 0.19374520D-01  T= 0.45082382D+02
 14 ESTIMATE=-0.97117708D-02  SE= 0.26221695D-01  T=-0.37037158D+00
 15 ESTIMATE=-0.87286209D-02  SE= 0.34641638D-01  T=-0.25196906D+00
 16 ESTIMATE=-0.68964024D-01  SE= 0.36058626D-01  T=-0.19125527D+01
 17 ESTIMATE= 0.12696217D+00  SE= 0.45099572D-01  T= 0.28151524D+01

PROCRUSTES LOADINGS OF FACTOR 2
 18 ESTIMATE= 0.79997681D-01  SE= 0.25240895D-01  T= 0.31693678D+01
 19 ESTIMATE=-0.54246929D-01  SE= 0.20959831D-01  T=-0.25881377D+01
 20 ESTIMATE=-0.47477411D-01  SE= 0.23867383D-01  T=-0.19892173D+01
 21 ESTIMATE= 0.42871722D-01  SE= 0.26414519D-01  T= 0.16230362D+01
 22 ESTIMATE= 0.95900949D+00  SE= 0.23092774D-01  T= 0.41528552D+02
 23 ESTIMATE= 0.80181990D+00  SE= 0.30998731D-01  T= 0.25866217D+02
 24 ESTIMATE= 0.79456209D+00  SE= 0.32902475D-01  T= 0.24149007D+02
 25 ESTIMATE= 0.61070170D+00  SE= 0.42773369D-01  T= 0.14277615D+02
```

6. Direct oblique rotation using the general symmetric family of quartic

criteria

6.1 DGQ1: Direct oblique solution with the general symmetric family of quartic criteria

SUBROUTINE DGQ1(M, MF, D, DR, PH, KAISER, AK1, AK2, AK3 AK4, IR, IC)

(1) Purpose

DGQ1 with CUBNE derives a direct oblique solution with the general symmetric family of quartic criteria. The rotation method includes the direct oblimin and primary parsimony methods as special cases (see Jennrich, 1973; Crawford, 1975; Clarkson & Jennrich, 1988; Browne & DuToit, 1992; Ogasawara, 1999b, 2000a).

(2) Arguments

For M, MF, KAISER, AK1, AK2, AK3, AK4, IR and IC, see Sections 1 and 2, and the remarks in this section.

D(IR, IC): An unrotated loading matrix of orthogonal factors/components (input).

The obliquely rotated loading matrix (output).

DR(IR, IC): The structure matrix for the obliquely rotated factors/components (output).

PH(IC, IC): The correlation matrix for the obliquely rotated factors/components (output).

(3) Remarks

An iterative computation method is used in DGQ1. The maximum number of iterations (IMAX=200) and the convergence criterion (EPS=0.000001) are sufficient for usual problems. They can be changed if necessary. The iteration stops when none of the rotated loadings is different from the corresponding loadings in the previous iteration stage by the amount specified by EPS.

DGQ1 uses the subroutine CUBNE which derives the solution of a cubic equation, in which the Newton-Raphson method is used with the maximum number of iterations (IMAX=200) and the convergence criterion (EPS=10**(-12)) for the absolute value of each element of the gradient vector. These values can also be changed. For the solution of a cubic equation, see e.g., Seino and Matsui (1971).

The followings are the values of AK1, AK2, AK3 and AK4 which correspond to typical oblique solutions.

(i) Oblimin method

AK1= -GAMMA, AK2=M, AK3=GAMMA, AK4=-M, where GAMMA is the oblimin weight (e.g., GAMMA=0 corresponds to the quartimin method; GAMMA=1 the covarimin method) and M is the number of observed variables.

(ii) Primary parsimony method

AK1=0, AK2=K1, AK3=K2, AK4=-K1-K2, where K1 and K2 are Crawford and Ferguson's (1970) weights. (The orthomax weight (W) is equivalent to M*K2/(K1+K2) in orthogonal rotation.) For the oblique (not orthogonal) varimax method, K1=M-1 and K2=1 (note that W=1 in this case). For other oblique primary parsimony criteria, see Crawford (1975, Table 1).

As was mentioned earlier, CUBNE is used in DGQ1.

6.2 QSEUN: The asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in factor analysis for unstandardized variables.

SUBROUTINE QSEUN(M, MF, N, D, PS, SR, C, KAISER, AK1, AK2, AK3 AK4, IR, IC, IH, ICOR)

(1) Purpose

QSEUN outputs the asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in factor analysis for unstandardized variables.

(2) Arguments

For M, MF, N, PS, KAISER, AK1, AK2, AK3, AK4, IR, IC, IH and ICOR, see Sections 1, 2 and 6.1.

D(IR, IC): The obliquely rotated loading matrix (input).

SR(IR, IC): The factor structure matrix of obliquely rotated factors (output).

C(IC, IC): The factor correlation matrix of obliquely rotated factors (input).

(3) Remarks

The subroutines REDGQ1 and INV are used in QSEUN. The function statements PM and QM are used in REDGQ1.

6.3 QSEST: The asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in factor analysis for standardized variables.

SUBROUTINE QSEST(M, MF, N, D, SR, C, KAISER, AK1, AK2, AK3 AK4, IR, IC, IH, ICOR)

(1) Purpose

QSEST outputs the asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in factor analysis for standardized variables.

(2) Arguments

For M, MF, N, KAISER, AK1, AK2, AK3, AK4, IR, IC, IH and ICOR, see Sections 1, 2 and 6.1.

D(IR, IC): The obliquely rotated loading matrix (input).

SR(IR, IC): The factor structure matrix of obliquely rotated factors (output).

C(IC, IC): The factor correlation matrix of obliquely rotated factors (input).

(3) Remarks

The subroutines/function statements REDGQ1, INV, CC and DC are used in QSEST. The function statements PM and QM are used in REDGQ1. For the scales (standard deviations of observed variables), see Section 1(2).

6.4 Example: Usage for DGQ1 and QSEST

(1) Program

```
#!/bin/csh
cat > $1.f << EOF
C
C     STANDARD ERRORS FOR OBLIQUE SOLUTIONS BY GENERAL QUARTIC CRITERIA
C
C     FOR STANDARDIZED VARIABLES (MODEL FOR CORRELATIONS)
C
C     <ROQSER>                    '98.8.24, '99.9.18 '99.10.7 2003.4.21
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*4 AN(20)
      DIMENSION D(20,3),SR(20,3),C(3,3)
      OPEN(5,FILE='$1.cards',STATUS='OLD')
   77 READ(5,120,END=99)(AN(I),I=1,20)
  120 FORMAT(20A4)
      WRITE(6,121)(AN(I),I=1,20)
  121 FORMAT(////,80('-')/'DIR.OBL.ROT. WITH GEN.QUART.CRI. :',
     &' MODEL FOR CORRELATIONS'//20A4)
C
```

```
      READ(5,101)M,MF,N,KAISER,AK1,AK2,AK3,AK4
  101 FORMAT(4I5,4F5.0)
      DO 1 I=1,MF
      READ(5,123)(D(J,I),J=1,M)
  123 FORMAT(12F5.0)
    1 WRITE(6,124)(D(J,I),J=1,M)
  124 FORMAT(/'ORTHOGONAL LOADINGS=',8F6.3)
      WRITE(6,102)N
  102 FORMAT(/,'N=',I4)
C
      IR=20
      IC=3
      IH=100
      CALL DGQ1(M,MF,D,SR,C,KAISER,AK1,AK2,AK3,AK4,IR,IC)
C
      ICOR=1
      CALL QSEST(M,MF,N,D,SR,C,KAISER,AK1,AK2,AK3,AK4,IR,IC,IH,ICOR)
C
      GO TO 77
   99 STOP
      END
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/dgq1 >> $1.f
cat ~/haru/redgq1 >> $1.f
cat ~/haru/qsest >> $1.f
cat > $1.cards << EOF
8 PHY. VAR. (HARMAN,1976,P.22,N=305) OBLIQUE-VARIMAX (=PARSIMAX): RAW
    8    2  305    0  0.0  7.0  1.0 -8.0
 .863 .926 .894 .857 .227 .189 .129 .273 --- NORMAL VARIMAX ---
 .293 .187 .185 .258 .927 .775 .753 .623 --- ORIGINAL ROTATED RESULTS---
EOF
f90 -o prog$1 $1.f >& $1.u
date >> $1.f
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
```

```
hostname >> $1.u
```

(2) Output (part)

```
SES OF SCALES, FACTOR COR. AND LOADINGS

SCALES (SET AT ONES WITHOUT LOSS OF GENERALITY)
      1  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      2  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      3  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      4  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      5  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      6  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      7  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02
      8  ESTIMATE= 0.10000000D+01  SE= 0.40555355D-01  T= 0.24657656D+02

FACTOR COR.:(2,1),(3,1),(3,2),...
      9  ESTIMATE= 0.35301265D+00  SE= 0.38606285D-01  T= 0.91439168D+01

ROTATED LOADINGS OF FACTOR 1 BY GEN.QUART.CRIT.
     10  ESTIMATE= 0.84228143D+00  SE= 0.18353051D-01  T= 0.45893266D+02
     11  ESTIMATE= 0.93068287D+00  SE= 0.12393660D-01  T= 0.75093467D+02
     12  ESTIMATE= 0.89757830D+00  SE= 0.14929040D-01  T= 0.60122974D+02
     13  ESTIMATE= 0.84338965D+00  SE= 0.18722858D-01  T= 0.45045988D+02
     14  ESTIMATE= 0.41983024D-01  SE= 0.22802916D-01  T= 0.18411253D+01
     15  ESTIMATE= 0.34283051D-01  SE= 0.33403438D-01  T= 0.10263330D+01
     16  ESTIMATE=-0.23932287D-01  SE= 0.35347184D-01  T=-0.67706346D+00
     17  ESTIMATE= 0.15440520D+00  SE= 0.43598275D-01  T= 0.35415439D+01

ROTATED LOADINGS OF FACTOR 2 BY GEN.QUART.CRIT.
     18  ESTIMATE= 0.16047257D+00  SE= 0.25209360D-01  T= 0.63655946D+01
     19  ESTIMATE= 0.37810546D-01  SE= 0.21122232D-01  T= 0.17900829D+01
     20  ESTIMATE= 0.41210875D-01  SE= 0.24167273D-01  T= 0.17052348D+01
     21  ESTIMATE= 0.12458593D+00  SE= 0.26554591D-01  T= 0.46916908D+01
     22  ESTIMATE= 0.93875960D+00  SE= 0.20234668D-01  T= 0.46393626D+02
     23  ESTIMATE= 0.78496549D+00  SE= 0.27924972D-01  T= 0.28109804D+02
     24  ESTIMATE= 0.77209008D+00  SE= 0.29760678D-01  T= 0.25943296D+02
```

```
  25  ESTIMATE= 0.61016439D+00  SE= 0.39101944D-01  T= 0.15604451D+02

SES & COR. FOR (1)DIRECT CONT.
 1 ESTIMATE= 3.1199 SE=  0.0605 T=51.5919 R= 1.000
 2 ESTIMATE= 2.5103 SE=  0.0775 T=32.4008 R= 0.163 1.000

SES & COR. FOR (2)SS OF STRUC.
 1 ESTIMATE= 3.7536 SE=  0.1178 T=31.8699 R= 1.000
 2 ESTIMATE= 3.2199 SE=  0.1708 T=18.8539 R= 0.884 1.000

SES & COR. FOR (3)ANTI-IMAGE.
 1 ESTIMATE= 2.7311 SE=  0.1235 T=22.1170 R= 1.000
 2 ESTIMATE= 2.1974 SE=  0.0975 T=22.5483 R= 0.717 1.000

SES & COR. FOR (4)JOINT CONT.
 1 ESTIMATE= 0.3209 SE=  0.0702 T= 4.5710 R= 1.000
```

(3) Note

For the four types of contributions (1) DIRECT CONT. through (4) JOINT CONT. in the example, see Ogasawara (1998a).

6.5 QSEUNP2A: The asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in principal component analysis for unstandardized variables.

SUBROUTINE QSEUNP2A(P, M, MF, N, KAISER, AK1, AK2, AK3 AK4, IR, IC, IH, ICOR)

(1) Purpose

QSEUNP2A outputs the asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in principal component analysis for unstandardized variables. (For the asymptotic standard errors, see Ogasawara, 2000c, 2002.)

(2) Arguments

For M, MF, N, KAISER, AK1, AK2, AK3, AK4, IR, IC, IH and ICOR, see Sections 1, 2 and 6.1.

P(IR, IR): An unbiased covariance matrix for observed variables (input).

(3) Remarks

Note that the input data set is not the rotated loading matrix as was for QSEUN, but is the covariance matrix for observed variables. The covariance matrix is necessary for the computation of the standard errors of the estimated component loadings as well as for the rotated solution.

The subroutines INV, EIGEN, DGQ1 and REDGQ1 are used in QSEUN. The function statements PM and QM are used in REDGQ1.

6.6 QSESTP2: The asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in principal component analysis for standardized variables.

SUBROUTINE QSESTP2(P, M, MF, N, KAISER, AK1, AK2, AK3 AK4, IR, IC, IH, ICOR)

(1) Purpose

QSESTP2 outputs the asymptotic standard errors of the direct oblique solution with the general symmetric family of quartic criteria in principal component analysis for standardized variables. (For the asymptotic standard errors, see Ogasawara, 2000c.)

(2) Arguments

For M, MF, N, KAISER, AK1, AK2, AK3, AK4, IR, IC, IH and ICOR, see Sections 1, 2 and 6.1.

P(IR, IR): A correlation matrix for observed variables (input).

(3) Remarks

Note that the input data set is not the rotated loading matrix as was for QSEST, but is the correlation matrix for observed variables. The correlation matrix is necessary for the computation of the standard errors of the estimated component loadings as well as for the rotated solution.

The subroutines/function statements REDGQ1, INV, CC and DC are used in QSEST. The function statements PM and QM are used in REDGQ1. For the scales (standard deviations of observed variables), see Section 1(2).

6.7 Example: Usage for QSESTP2

(1) Program

```
#!/bin/csh
cat > $1.f << EOF
C
C     A CONCISE METHOD FOR THE STANDARD ERRORS
```

```
C      OF THE OBLIQUELY ROTATED SOLUTIONS BY GENERAL QUARTIC CRITERIA
C      FOR PCA FOR STANDARDIZED VARIABLES
C
C      <ROQSERP2>                                2000.2.23-2003.4.18
C
C
       IMPLICIT REAL*8(A-H,O-Z)
       CHARACTER*4 AN(20)
       DIMENSION R(20,20)
       OPEN(5,FILE='$1.cards')
     1 READ(5,102,END=99)(AN(I),I=1,20)
   102 FORMAT(20A4)
       WRITE(6,103)(AN(I),I=1,20)
   103 FORMAT(////,80('-')/20A4)
       READ(5,104)M,MF,N,KAISER,AK1,AK2,AK3,AK4
   104 FORMAT(4I5,4F5.0)
       WRITE(6,105)N
   105 FORMAT(//,'N=',I4/)
C
       DO 6 I=1,M
       READ(5,107)(R(I,J),J=1,I)
   107 FORMAT(12F5.0)
       WRITE(6,117)(R(I,J),J=1,I)
   117 FORMAT('COR=',8F8.4)
     6 CONTINUE
       DO 8 I=1,M
       DO 8 J=1,I
       R(J,I)=R(I,J)
     8 CONTINUE
C
C      PCA
       IR=20
       IC=MF
       IH=M**2+(M**2-M)/2+(MF**2-MF)+2*M
       ICOR=1
C
```

```
      CALL QSESTP2(R,M,MF,N,KAISER,AK1,AK2,AK3,AK4,IR,IC,IH,ICOR)
C
      GO TO 1
   99 STOP
      END
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/eigen >> $1.f
cat ~/haru/dgq1 >> $1.f
cat ~/haru/redgq1 >> $1.f
cat ~/haru/qsestp2 >> $1.f
cat > $1.cards << EOF
6 SCHOOL SUBJECTS (LAWLEY & MAXWELL,1971,P66):  RAW QUARTIMIN
    6    2  220    0  0.0  1.0  0.0 -1.0
 1.
 .439 1.
 .410 .351 1.
 .288 .354 .164 1.
 .329 .320 .190 .595 1.
 .248 .329 .181 .470 .464 1.
6 SCHOOL SUBJECTS (LAWLEY & MAXWELL,1971,P66):  NORMAL QUARTIMIN
    6    2  220    1  0.0  1.0  0.0 -1.0
 1.
 .439 1.
 .410 .351 1.
 .288 .354 .164 1.
 .329 .320 .190 .595 1.
 .248 .329 .181 .470 .464 1.
EOF
f90 -o prog$1 $1.f >& $1.u
date > $1.u
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
hostname >> $1.u
```

(2) Output (part)

```
SES OF ROTATED COMPONENT COR. & LOADINGS
BY GENERAL QUARTIC CRITERIA FOR STANDARDIZED VARIABLES

COMPONENT COR.: (2,1),(3,1),(3,2),...
  1  ESTIMATE= 0.34492748D+00  SE= 0.62221191D-01  T= 0.55435692D+01

OBLIQUELY ROTATED LOADINGS OF COMPONENT  1
  2  ESTIMATE= 0.10701937D+00  SE= 0.96187584D-01  T= 0.11126111D+01
  3  ESTIMATE= 0.25511809D+00  SE= 0.11908782D+00  T= 0.21422685D+01
  4  ESTIMATE=-0.14235380D+00  SE= 0.35682947D-01  T=-0.39894071D+01
  5  ESTIMATE= 0.85110911D+00  SE= 0.32486182D-01  T= 0.26199111D+02
  6  ESTIMATE= 0.82508222D+00  SE= 0.37868912D-01  T= 0.21787852D+02
  7  ESTIMATE= 0.76245135D+00  SE= 0.57400932D-01  T= 0.13282909D+02

OBLIQUELY ROTATED LOADINGS OF COMPONENT  2
  8  ESTIMATE= 0.75320267D+00  SE= 0.60658583D-01  T= 0.12417083D+02
  9  ESTIMATE= 0.61974846D+00  SE= 0.92355657D-01  T= 0.67104548D+01
 10  ESTIMATE= 0.85905223D+00  SE= 0.34803825D-01  T= 0.24682696D+02
 11  ESTIMATE=-0.15183310D-01  SE= 0.55895363D-01  T=-0.27163809D+00
 12  ESTIMATE= 0.22890147D-01  SE= 0.61759579D-01  T= 0.37063315D+00
 13  ESTIMATE= 0.89959228D-02  SE= 0.91621649D-01  T= 0.98185558D-01

SCALES (SET AT ONES WITHOUT LOSS OF GENERALITY)
 38  ESTIMATE= 0.10000000D+01  SE= 0.47781848D-01  T= 0.20928450D+02
 39  ESTIMATE= 0.10000000D+01  SE= 0.47781848D-01  T= 0.20928450D+02
 40  ESTIMATE= 0.10000000D+01  SE= 0.47781848D-01  T= 0.20928450D+02
 41  ESTIMATE= 0.10000000D+01  SE= 0.47781848D-01  T= 0.20928450D+02
 42  ESTIMATE= 0.10000000D+01  SE= 0.47781848D-01  T= 0.20928450D+02
 43  ESTIMATE= 0.10000000D+01  SE= 0.47781848D-01  T= 0.20928450D+02


COR. FOR ROTATED COMPONENT COR. & LOADINGS
BY GENERAL QUARTIC CRITERIA FOR STANDARDIZED VARIABLES

COMPONENT COR.: (2,1),(3,1),(3,2),...
 1.00
```

```
OBLIQUELY ROTATED LOADINGS OF COMPONENT  1
-0.38 1.00
-0.45 0.32 1.00
 0.54-0.84-0.16 1.00
 0.02-0.02 0.03 0.08 1.00
 0.07 0.05-0.16-0.08-0.07 1.00
 0.16-0.13 0.06 0.15-0.38-0.38 1.00

OBLIQUELY ROTATED LOADINGS OF COMPONENT  2
 0.27-0.88-0.14 0.70 0.02-0.12 0.17 1.00
 0.34-0.25-0.92-0.04-0.04 0.19-0.09 0.08 1.00
-0.10 0.70 0.48-0.36 0.05 0.03-0.04-0.53-0.40 1.00
 0.15-0.01-0.14-0.04-0.79 0.23 0.48-0.01 0.14-0.06 1.00
 0.02-0.04 0.05 0.07 0.22-0.82 0.45 0.10-0.09-0.03-0.34 1.00
-0.08 0.10-0.10-0.10 0.53 0.50-0.83-0.15 0.13 0.05-0.58-0.54 1.00

SCALES(ROWS) VS. COMPONENT COR. & LOADINGS(COLUMNS)
 0.19 0.05-0.03-0.03 0.04 0.00 0.05 0.07 0.06 0.19 0.01 0.05-0.02
 0.18 0.02 0.06-0.03 0.05 0.05 0.04 0.03 0.04 0.18 0.02-0.02 0.01
 0.15-0.07-0.06 0.10 0.05 0.02 0.01 0.12 0.09 0.13-0.02 0.01 0.02
 0.10 0.08 0.11-0.02 0.18 0.16 0.14-0.05-0.08 0.15 0.05-0.02-0.04
 0.13 0.10 0.06-0.03 0.17 0.17 0.14-0.07-0.05 0.15 0.00 0.04-0.03
 0.10 0.04 0.08 0.01 0.13 0.11 0.15-0.02-0.06 0.11-0.03-0.03 0.04

SCALES
 1.00
 0.19 1.00
 0.17 0.12 1.00
 0.08 0.13 0.03 1.00
 0.11 0.10 0.04 0.35 1.00
 0.06 0.11 0.03 0.22 0.22 1.00
```

7. Promax rotation

7.1 PRMSEUN: Promax solution with its asymptotic standard errors in factor analysis for unstandardized variables

SUBROUTINE PRMSEUN(M, MF, N, D, ND, PSI, W, INRM, IQ, IR, IC, IH, ICOR)

(1) Purpose

PRMSEUN with OTMAX and PRMX outputs the promax solution with its asymptotic standard errors in factor analysis for unstandardized variables from the unrotated solution with the parameter pattern of the solution given by FAOC. For the promax solution and the algorithms for the standard errors employed by PRMSEUN, see Hendrickson and White (1964) and Ogasawara (1998b), respectively.

(2) Arguments

For M, MF, N, PSI, W, INRM, IR, IC, IH and ICOR, see Sections 1 and 2.

D(IR, IC): The unrotated loading matrix for unstandardized variables with the parameter pattern given by FAOC. That is, D(,) should have (MF**2-MF)/2 zeroes in appropriate positions (input).

ND(IR, IC): The 0/1 pattern matrix corresponding to the unrotated loading matrix, where 1's indicate that the corresponding elements are free parameters and 0's indicate that the corresponding elements are fixed at zero (input).

IQ: The power by which orthomax rotated loadings are raised. IQ should be odd numbers (3, 5,…). IQ=3 is recommended.

(3) Remarks

The subroutine FAOC may be used to obtain the unrotated loading matrix with the parameter pattern described above. Otherwise, an unrotated orthogonal loading matrix should be rotated to that with the same parameter pattern given by FAOC.

In PRMSEUN and the following PRMSEST, the unrotated loading matrix is linearly transformed to fit the target matrix which has been constructed from an orthomax rotated loading matrix. The transformation optimizes a criterion of unrestricted least squares. After the transformation, the transformed factors are scaled to have unit variances.

The subroutines OTMAX, INV and PRMX are used in PRMSEUN.

7.2 PRMSEST: Promax solution with its asymptotic standard errors in factor analysis for standardized variables

SUBROUTINE PRMSEST(M, MF, N, D, ND, W, INRM, IQ, IR, IC, IH, ICOR)

(1) Purpose

PRMSEST with OTMAX and PRMX outputs the promax solution with its asymptotic standard errors for standardized variables from the unrotated solution with the parameter pattern of the solution given by FAOC. For the promax solution and the algorithms employed by PRMSEUN for the standard errors, see Hendrickson and White (1964) and Ogasawara (1998b), respectively (see also Yung & Hayashi, 2001).

(2) Arguments

For M, MF, N, W, INRM, IR, IC, IH and ICOR, see Sections 1 and 2.

D(IR, IC): The unrotated loading matrix in factor analysis for standardized variables with the parameter pattern given by FAOC. That is, D(,) should have (MF**2-MF)/2 zeroes in appropriate positions (input).

ND(IR, IC): The 0/1 pattern matrix corresponding to the unrotated loading matrix, where 1's indicate that the corresponding elements are free parameters and 0's indicate that the corresponding elements are fixed at zero (input).

IQ: The power by which orthomax rotated loadings are raised. IQ should be odd numbers (3, 5,…). IQ=3 is recommended.

(3) Remarks

The subroutines OTMAX, INV and PRMX are used in PRMSEST. See also Remarks in 7.1. For the scales (standard deviations of observed variables), see Section 1(2).

7.3 Example: Usage for FAOC and PRMSEST

(1) Program

```
#!/bin/csh
cat > $1.f << EOF
C
C     STANDARD ERRORS OF PROMAX SOLUTION FOR STANDARDIZED VARIABLES
C
C     <ROPRMSER>                                    '99.9.26 2003.4.21
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*4 AN(20)
      DIMENSION P(20,20),S(20,20),D(20,3),ND(20,3),PSI(20)
      OPEN(5,FILE='$1.cards',STATUS='OLD')
  122 READ(5,123,END=99)(AN(I),I=1,20)
  123 FORMAT(20A4)
```

```
      WRITE(6,124)(AN(I),I=1,20)
  124 FORMAT(////,80('-'),/20A4)
      READ(5,125)W,INRM,M,MF,N,IQ,ICOR
  125 FORMAT(F5.0,4I5/2I5)
      WRITE(6,126)N
  126 FORMAT(//,'N=',I4)
C
      DO 9 I=1,M
      READ(5,101)(S(I,J),J=1,I)
  101 FORMAT(12F5.0)
      DO 9 J=1,I
      S(J,I)=S(I,J)
    9 CONTINUE
C
      DO 63 I=1,MF
      READ(5,206)(ND(J,I),J=1,M)
  206 FORMAT(40I2)
   63 WRITE(6,118)(ND(J,I),J=1,M)
  118 FORMAT(/,'PATTERN OF FACTOR LOADINGS=',12I4)
      DO 40 I=1,MF
      READ(5,117)(D(J,I),J=1,M)
   40 WRITE(6,119)(D(J,I),J=1,M)
  119 FORMAT(/,'INITIAL VALUE=',12F5.2)
  117 FORMAT(12F4.1)
      READ(5,117)(PSI(I),I=1,M)
C
      IR=20
      IC=3
      IH=150
      CALL FAOC(M,MF,N,S,P,D,ND,PSI,IR,IC,IH)
C
      CALL PRMSEST(M,MF,N,D,ND,W,INRM,IQ,IR,IC,IH,ICOR)
C
      GO TO 122
   99 CONTINUE
      STOP
```

```
      END
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/faoc >> $1.f
cat ~/haru/prmsest >> $1.f
cat ~/haru/otmax >> $1.f
cat ~/haru/prmx >> $1.f
cat > $1.cards << EOF
8 PHYSICAL VARIABLES (HARMAN,1976,P22)
  1.0    1    8    2  305
    3    1
 1.
 .846 1.
 .805 .881 1.
 .859 .826 .801 1.
 .473 .376 .380 .436 1.
 .398 .326 .319 .329 .762 1.
 .301 .277 .237 .327 .730 .583 1.
 .382 .415 .345 .365 .629 .577 .539 1.
 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1
 .91 .94 .91 .89 .51 .43 .36 .46
 .00-.12-.11-.03 .81 .67 .67 .50
 .17 .11 .17 .20 .09 .36 .42 .54  ---- PSI
EOF
f90 -o prog$1 $1.f >& $1.u
date >> $1.u
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
hostname >> $1.u
```

(2) Output (part)

```
SES OF PROMAX-ROTATED LOADINGS AND FACTOR COR.

PROMAX LOADINGS OF FACTOR 1
  1  ESTIMATE=  0.8656059D+00  SE=  0.1911371D-01  T=  0.4528717D+02
```

```
  2  ESTIMATE=  0.9631749D+00  SE=  0.1301148D-01  T=  0.7402502D+02
  3  ESTIMATE=  0.9286606D+00  SE=  0.1563774D-01  T=  0.5938587D+02
  4  ESTIMATE=  0.8684805D+00  SE=  0.1952984D-01  T=  0.4446941D+02
  5  ESTIMATE= -0.1319778D-03  SE=  0.2609520D-01  T= -0.5057551D-02
  6  ESTIMATE= -0.7152759D-03  SE=  0.3445014D-01  T= -0.2076264D-01
  7  ESTIMATE= -0.6065070D-01  SE=  0.3484972D-01  T= -0.1740350D+01
  8  ESTIMATE=  0.1322403D+00  SE=  0.4619724D-01  T=  0.2862515D+01

PROMAX LOADINGS OF FACTOR 2
  9  ESTIMATE=  0.9117541D-01  SE=  0.2689851D-01  T=  0.3389608D+01
 10  ESTIMATE= -0.4113021D-01  SE=  0.2120378D-01  T= -0.1939758D+01
 11  ESTIMATE= -0.3485358D-01  SE=  0.2398950D-01  T= -0.1452868D+01
 12  ESTIMATE=  0.5426381D-01  SE=  0.2761098D-01  T=  0.1965299D+01
 13  ESTIMATE=  0.9544620D+00  SE=  0.2261183D-01  T=  0.4221073D+02
 14  ESTIMATE=  0.7980097D+00  SE=  0.3035495D-01  T=  0.2628928D+02
 15  ESTIMATE=  0.7899861D+00  SE=  0.3197642D-01  T=  0.2470527D+02
 16  ESTIMATE=  0.6095725D+00  SE=  0.4289213D-01  T=  0.1421176D+02

FACTOR COR.:(2,1),(3,1),(3,2),...
 17  ESTIMATE=  0.4602394D+00  SE=  0.4095061D-01  T=  0.1123889D+02
```

8. Orthoblique rotation

8.1 HKOB: Case II orthoblique solution

SUBROUTINE HKOB(M, MF, D, D4, PH, DI, S1, S1A, HS1, Q, EM, T3, PW, W, INRM, IR, IC)

(1) Purpose

HKOB derives the case II orthoblique solution using the orthomax method for the first-stage orthogonal rotation. The solution includes, as special cases, the independent cluster solution and the proportional solution (see Harris & Kaiser, 1964; Ogasawara, 2000b).

(2) Arguments

For M, MF, IR and IC, see Sections 1 and 2.

D(IR, IC): An unrotated loading matrix of orthogonal factors (input).

The orthomax-rotated loading matrix in the first stage of the orthoblique rotation (output). Note that the orthomax solution in HKOB is different from the usual orthomax solution in that the corresponding unrotated matrix in HKOB

is a rescaled loading matrix.
D4(IR, IC): The orthoblique-rotated loading matrix (output).
PH(IC, IC): The factor correlation matrix for the orthoblique solution (output).
PW: The power used in the orthoblique solution (0<=PW<1; input).

When PW=0, we have the independent cluster solution. When PW=0.5, we have the proportional solution.

W: The orthomax weight for the orthomax solution in the first stage of the orthoblique solution.
INRM: When Kaiser's normalization is employed in the orthomax rotation in the first stage of the orthoblique solution, INRM=1. Otherwise, INRM=0.

The arguments, DI, S1, S1A, HS1, Q, EM and T3, are for output, which may be used for the input of HKSEUN and/or HKSEST. Usually these arguments will not directly be used by users, but should be specified as arrays in the DIMENSION statement of the main routine which calls HKOB as follows: DI(IC), S1(IC), S1A(IC), HS1(IR), Q(IR, IR), EM(IR) and T3(IC, IC) with appropriate values of IR and IC.

(3) Remarks

The subroutine EIGEN and INV are used in HKOB.

8.2 HKSEUN2: The asymptotic standard errors of the orthoblique solution in factor analysis for unstandardized variables

SUBROUTINE HKSEUN2(M, MF, N, D, D4, PH, DI, S1, HS1, PSI, Q, EM, T3, PW, W, INRM, IR, IC, IH, ICOR)

(1) Purpose

HKSEUN2, using the results of the preceding HKOB, outputs the asymptotic standard errors of the case II orthoblique solution in factor analysis for unstandardized variables.

(2) Arguments

The arguments are all for input and are the same as the corresponding arguments in HKOB except for the additional arguments N, PSI and ICOR and no use of S1A. For N, PSI and ICOR, see Section 2.

(3) Remarks

The subroutine HKOB should be called by the main routine before HKSEUN is called. The subroutine INV is used in HKSEUN.

8.3 HKSEST2: The asymptotic standard errors of the orthoblique solution

in factor analysis for standardized variables

SUBROUTINE HKSEST2(M, MF, N, D, D4, PH, DI, S1, HS1, Q, EM, T3, PW, W, INRM, IR, IC, IH, ICOR)

(1) Purpose

HKSEST2, using the results of the preceding HKOB, outputs the asymptotic standard errors of the case II orthoblique solution for standardized variables.

(2) Arguments

The arguments are all for input and are the same as the corresponding arguments in HKOB except for the additional arguments N and ICOR and no use of S1A. For N and ICOR, see Section 2. Note that the argument PSI, which was included in HKSEUN2, is not required in HKSEST2.

(3) Remarks

The subroutine HKOB should be called by the main routine before HKSEST2 is called. The subroutine INV is used in HKSEST2. For the scales (standard deviations of observed variables), see Section 1(2) Remarks.

8.4 Example: Usage for FAOC, HKOB and HKSEST2

(1) Program

```
#!/bin/csh
cat > $1.f << EOF
C
C     STANDARD ERRORS FOR HARRIS-KAISER CASE II ORTHOBLIQUE SOLUTION
C       FOR STANDARDIZED VARIABLES
C
C     <ROHKSER>                               '99.10.8 2003.4.21
C
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*4 AN(20)
      DIMENSION S1(3),S1A(3),P(20,20),S(20,20),Q(20,20),EM(20),T3(3,3),
     & D(20,3),ND(20,3),PSI(20),D4(20,3),PH(3,3),DI(3),HS1(20)
      OPEN(5,FILE='$1.cards')
  122 READ(5,123,END=99)(AN(I),I=1,20)
  123 FORMAT(20A4)
      WRITE(6,124)(AN(I),I=1,20)
  124 FORMAT(////,80('-'),/20A4)
      READ(5,125)PW,W,INRM,M,MF,N
```

```
  125 FORMAT(2F5.0,4I5)
      WRITE(6,126)N
  126 FORMAT(//,'N=',I4)
C
      DO 9 I=1,M
      READ(5,101)(S(I,J),J=1,I)
  101 FORMAT(12F5.0)
      DO 9 J=1,I
      S(J,I)=S(I,J)
    9 CONTINUE
C
      DO 63 I=1,MF
      READ(5,206)(ND(J,I),J=1,M)
  206 FORMAT(40I2)
   63 WRITE(6,118)(ND(J,I),J=1,M)
  118 FORMAT(/,'PATTERN OF FACTOR LOADINGS=',12I4)
      DO 40 I=1,MF
      READ(5,117)(D(J,I),J=1,M)
   40 WRITE(6,119)(D(J,I),J=1,M)
  119 FORMAT(/,'INITIAL VALUE=',12F5.2)
  117 FORMAT(12F4.1)
      READ(5,117)(PSI(I),I=1,M)
C
      IR=20
      IC=3
      IH=150
      CALL FAOC(M,MF,N,S,P,D,ND,PSI,IR,IC,IH)
C
      CALL HKOB(M,MF,D,D4,PH,DI,S1,S1A,HS1,Q,EM,T3,PW,W,INRM,IR,IC)
C
      ICOR=1
      CALL HKSEST2(M,MF,N,D,D4,PH,DI,S1,S1A,HS1,Q,EM,T3,
     &             PW,W,INRM,IR,IC,IH,ICOR)
C
      GO TO 122
   99 STOP
```

```
      END
C
EOF
cat ~/haru/inv >> $1.f
cat ~/haru/faoc >> $1.f
cat ~/haru/otmax >> $1.f
cat ~/haru/eigen >> $1.f
cat ~/haru/hkob >> $1.f
cat ~/haru/hksest2 >> $1.f
cat > $1.cards << EOF
8 PHYSICAL VARIABLES (HARMAN,1976,P22)
  0.5  1.0    1    8    2  305
 1.
 .846 1.
 .805 .881 1.
 .859 .826 .801 1.
 .473 .376 .380 .436 1.
 .398 .326 .319 .329 .762 1.
 .301 .277 .237 .327 .730 .583 1.
 .382 .415 .345 .365 .629 .577 .539 1.
 1 1 1 1 1 1 1 1
 0 1 1 1 1 1 1 1
 .91 .94 .91 .89 .51 .43 .36 .46
 .00-.12-.11-.03 .81 .67 .67 .50
 .17 .11 .17 .20 .09 .36 .42 .54  ---- PSI
EOF
f90 -o prog$1 $1.f >& $1.u
date >> $1.u
(time prog$1) >>& $1.u
cat $1.cards >> $1.u
hostname >> $1.u
```

(2) Output (part)

```
SES OF ROTATED LOADINGS AND FACTOR COR.

 BY HARRIS-KAISER ORTHOBLIQUE METHOD: POWER= 0.50
```

```
ORTHOBLIQUE LOADINGS OF FACTOR 1
ESTIMATE=  0.8482894D+00  SE=  0.1717954D-01  T=  0.4937788D+02
ESTIMATE=  0.9269568D+00  SE=  0.1203946D-01  T=  0.7699323D+02
ESTIMATE=  0.8943113D+00  SE=  0.1435109D-01  T=  0.6231662D+02
ESTIMATE=  0.8466826D+00  SE=  0.1756855D-01  T=  0.4819309D+02
ESTIMATE=  0.1133353D+00  SE=  0.2572626D-01  T=  0.4405434D+01
ESTIMATE=  0.9417263D-01  SE=  0.3367484D-01  T=  0.2796528D+01
ESTIMATE=  0.3523291D-01  SE=  0.3529325D-01  T=  0.9982903D+00
ESTIMATE=  0.2004028D+00  SE=  0.4151600D-01  T=  0.4827122D+01

ORTHOBLIQUE LOADINGS OF FACTOR 2
ESTIMATE=  0.1794005D+00  SE=  0.2540087D-01  T=  0.7062768D+01
ESTIMATE=  0.6235189D-01  SE=  0.2274027D-01  T=  0.2741915D+01
ESTIMATE=  0.6474140D-01  SE=  0.2511820D-01  T=  0.2577470D+01
ESTIMATE=  0.1441684D+00  SE=  0.2647545D-01  T=  0.5445362D+01
ESTIMATE=  0.9188852D+00  SE=  0.2070329D-01  T=  0.4438352D+02
ESTIMATE=  0.7682004D+00  SE=  0.2772315D-01  T=  0.2770970D+02
ESTIMATE=  0.7541317D+00  SE=  0.2957210D-01  T=  0.2550145D+02
ESTIMATE=  0.6008574D+00  SE=  0.3783087D-01  T=  0.1588273D+02

FACTOR COR.:(2,1),(3,1),(3,2),...
ESTIMATE=  0.2577581D+00  SE=  0.2940453D-01  T=  0.8765933D+01
```

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